Since a lot of time and effort has gone into Pardiso version 8.0, please cite the following publications if you are using it for your own research:

References


Note: This document briefly describes the interface to the shared-memory multiprocessing parallel direct and multi-recursive iterative solvers in Pardiso 8.0.\footnote{Please note that this version is a significant extension compared to Intel’s MKL version and that these improvements and features are not available in Intel’s MKL 9.3 release or later releases.} A discussion of the algorithms used in Pardiso and more information on the solver can be found at \url{http://www.panua.ch/pardiso}
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1 Introduction

The package Pardiso is a high-performance, robust, memory-efficient and easy to use software for solving large sparse symmetric and nonsymmetric linear systems of equations on shared-memory architectures. The solver uses a combination of left- and right-looking Level-3 BLAS supernode techniques [15]. In order to improve sequential and parallel sparse numerical factorization performance, the algorithms are based on a Level-3 BLAS update, and pipelining parallelism is exploited with a combination of left- and right-looking supernode techniques [9, 11, 12, 14]. The parallel pivoting methods allow complete supernode pivoting in order to balance numerical stability and scalability during the factorization process. For sufficiently large problem sizes, numerical experiments demonstrate that the scalability of the parallel algorithm is nearly independent of the shared-memory multiprocessing architecture and a speedup of up to seven using eight processors has been observed. The approach is based on OpenMP [4] directives, and has been successfully tested on almost all shared-memory parallel systems.

Pardiso calculates the solution of a set of sparse linear equations with multiple right-hand sides,

\[ AX = B, \]

using a parallel \( LU, LDL^T \) or \( LL^T \) factorization, where \( A \) and \( X, B \) are \( n \) by \( n \) and \( n \) by \( nrhs \) matrices respectively. Pardiso supports, as illustrated in Figure 1, a wide range of sparse matrix types and computes the solution of real or complex, symmetric, structurally symmetric or nonsymmetric, positive definite, indefinite or Hermitian sparse linear systems of equations on shared-memory multiprocessing architectures.

Pardiso 8.0

![Sparse matrices that can be solved with Pardiso Version 8.0](image)

Pardiso 8.0 has the unique feature among all solvers that it can compute the exact bit-identical solution on multicore and cluster of multicore architectures (see release notes for more details).

1.1 Supported Matrix Types

Pardiso performs the following analysis steps depending on the structure of the input matrix \( A \).

1.1.1 Symmetric Matrices

The solver first computes a symmetric fill-in reducing permutation \( P \) based on either the minimum degree algorithm [6] or the nested dissection algorithm from the METIS package [2], followed by the parallel left-right looking numerical Cholesky factorization [15] \( PAP^T = LL^T \) or \( PAP^T = LDL^T \) for symmetric, indefinite matrices. The solver uses diagonal pivoting or \( 1 \times 1 \) and \( 2 \times 2 \) Bunch-Kaufman pivoting for symmetric indefinite matrices, and an approximation of \( X \) is found by forward and backward substitution and iterative refinement. The coefficient matrix is perturbed whenever numerically acceptable \( 1 \times 1 \) and \( 2 \times 2 \) pivots cannot be found within a diagonal supernode block. One or two passes of iterative refinement may be required to
correct the effect of the perturbations. This restricting notion of pivoting with iterative refinement is effective for highly indefinite symmetric systems. Furthermore, the accuracy of this method is for a large set of matrices from different applications areas as accurate as a direct factorization method that uses complete sparse pivoting techniques [13]. Another possibility to improve the pivoting accuracy is to use symmetric weighted matchings algorithms. These methods identify large entries in A that, if permuted close to the diagonal, permit the factorization process to identify more acceptable pivots and proceed with fewer pivot perturbations. The methods are based on maximum weighted matchings and improve the quality of the factor in a complementary way to the alternative idea of using more complete pivoting techniques. Additionally, for real symmetric indefinite matrices the inertia is computed.

1.1.2 Structurally Symmetric Matrices

The solver first computes a symmetric fill-in reducing permutation P based on the parallel numerical factorization of $PAP^T = QLU^T$. The solver uses partial pivoting in the supernodes and an approximation of X is found by forward and backward substitution and iterative refinement.

1.1.3 Nonsymmetric Matrices

The solver first computes a non-symmetric permutation $P_{MPS}$ and scaling matrices $D_r$ and $D_c$ with the aim to place large entries on the diagonal, which enhances greatly the reliability of the numerical factorization process [1]. In the next step the solver computes a fill-in reducing permutation $P$ based on the matrix $P_{MPS}A + (P_{MPS}A)^T$ followed by the parallel numerical factorization $QLUR = A_2$, $A_2 = PP_{MPS}D_rAD_rP$ with supernode pivoting matrices $Q$ and $R$ and $P = P(P_{MPS})$ to keep sufficiently large parts of the cycles of $P_{MPS}$ in one diagonal block. When the factorization algorithm reaches a point where it cannot factorize the supernodes with this pivoting strategy, it uses a pivoting perturbation strategy similar to [3]. The magnitude of the potential pivot is tested against a constant threshold of $\alpha = \epsilon \cdot ||A_2||_{\infty}$, where $\epsilon$ is the machine precision, and $||A_2||_{\infty}$ is the $\infty$-norm of the scaled and permuted matrix A. Any tiny pivots $l_{ii}$ encountered during elimination are set to the sign($l_{ii}$) $\epsilon \cdot ||A_2||_{\infty}$ — this trades off some numerical stability for the ability to keep pivots from getting too small. Although many failures could render the factorization well-defined but essentially useless, in practice it is observed that the diagonal elements are rarely modified for a large class of matrices. The result of this pivoting approach is that the factorization is, in general, not exact and iterative refinement may be needed.

1.2 Computing Selected Elements of the Inverse $A^{-1}$: PARDISO-INV

The parallel selected inversion method is an efficient way for computing e.g. the diagonal elements of the inverse of a sparse matrix. For a symmetric matrix A, the selected inversion algorithm first constructs the $LDL^T$ factorization of A, where L is a block lower diagonal matrix called the Cholesky factor, and D is a block diagonal matrix. In the second step, the selected inversion algorithm only computes all the elements $A_{ij}^{-1}$ such that $L_{ij} \neq 0$ in exact arithmetic. The computational scaling of the selected inversion algorithm is only proportional to the number of nonzero elements in the Cholesky factor L, even though $A^{-1}$ might be a full matrix. The selected inversion algorithm scales as $O(N)$ for quasi-1D systems, $O(N^{1.5})$ for quasi-2D systems, $O(N^2)$ for 3D systems. The selected inversion method achieves significant improvement over the traditional forward/backward method at all dimensions.

The parallel threaded implementation [3] of the selected inversion algorithm, called PARDISO-INV, is available for sparse symmetric or unsymmetric matrices, and it is fully integrated into the Pardiso package. In addition, it includes pivoting to maintain accuracy during both sparse inversion steps.

1.3 Computing the Schur-complement: PARDISO-SCHUR

The Schur-complement framework is a general way to solve linear systems. Much work has been done in this area; see, for example [8] and the references therein. Let $Ax = b$ be the system of interest. Suppose A has the form
A traditional approach to compute the exact Schur complement \( S = A_{22} - A_{21}A_{11}^{-1}A_{12} \) by using off-the-shelf linear solvers such as Pardiso is (i) to factor \( A_{11} \) and (ii) to solve for each right-hand side of \( A_{12} \) (in fact for small blocks of right-hand sides). However, this approach has two important drawbacks: (1) the sparsity of the right-hand sides \( A_{12} \) and \( A_{21} \) is not exploited, and (2) the calculations limit the efficient use of multicore shared-memory environments because it is well known that the triangular solves do not scale very well with the number of cores. In [7] we developed a fast and parallel reformulation of the sparse linear algebra calculations that circumvents the issues in the Schur complement computations. The idea is to leverage the good scalability of parallel factorizations as an alternative method of building the Schur complement based on forward/backward substitutions. We employ an incomplete augmented factorization technique that solves the sparse linear systems with multiple right-hand sides at once using an incomplete sparse factorization of the matrix \( A \).

The parallel threaded implementation of the Schur-complement algorithm, called \texttt{PARDISO-SCHUR} is available for sparse symmetric or unsymmetric matrices, and it is fully integrated into the Pardiso package. In addition, it includes pivoting to maintain accuracy during the Schur complement computations.

This solver is only available for real symmetric indefinite matrices.

### 1.4 Multi-Recursive Iterative Sparse Solver for Symmetric Indefinite Linear Systems

Pardiso includes a novel preconditioning solver [10]. Our preconditioning approach for symmetric indefinite linear system is based on maximum weighted matchings and algebraic multilevel incomplete \( LDL^T \) factorizations. These techniques can be seen as a complement to the alternative idea of using more complete pivoting techniques for the highly ill-conditioned symmetric indefinite matrices. In considering how to solve the linear systems in a manner that exploits sparsity as well as symmetry, we combine a diverse range of topics that includes preprocessing the matrix with symmetric weighted matchings, solving the linear system with Krylov subspace methods, and accelerating the linear system solution with multilevel preconditioners based upon incomplete factorizations.

### 1.5 Direct-Iterative Preconditioning for Nonsymmetric Linear Systems

The solver also allows a combination of direct and iterative methods [16] in order to accelerate the linear solution process for transient simulation. Many applications of sparse solvers require solutions of systems with gradually changing values of the nonzero coefficient matrix, but the same identical sparsity pattern. In these applications, the analysis phase of the solvers has to be performed only once and the numerical factorizations are the important time-consuming steps during the simulation. Pardiso uses a numerical factorization \( A = LU \) for the first system and applies these exact factors \( L \) and \( U \) for the next steps in a preconditioned Krylov-Subspace iteration. If the iteration does not converge, the solver will automatically switch back to the numerical factorization. This method can be applied for nonsymmetric matrices in Pardiso and the user can select the method using only one input parameter. For further details see the parameter description \texttt{IPARM(4), IPARM(20)}.

### 2 Specification

The library is available on 32-bit and 64-bit architectures. The library is based on Fortran and C source code and the top level driver routines \texttt{PARDISO} and \texttt{PARDISOINIT} assume the following interface.

#### On 32-bit architectures:

```
SUBROUTINE PARDISOINIT(PT, MTYPE, SOLVER, IPARM, DPARM, ERROR)
INTEGER*4 PT(64), MTYPE, SOLVER, IPARM(64), DPARM, ERROR
```
### Figure 2: Illustration of the two input compressed sparse row formats of an $8 \times 8$ general sparse nonsymmetric linear system and the upper triangular part of a symmetric linear system for the direct solver Pardiso.

The algorithms in Pardiso require JA to be increasingly ordered per row and the presence of the diagonal element per row for any symmetric or structurally symmetric matrix. The nonsymmetric matrices need no diagonal elements in the Pardiso solver.

```plaintext
REAL*8 DPARM(64)

and

SUBROUTINE PARDISO(PT, MAXFCT, MNUM, MTYPE, PHASE, N, A, IA, JA,
                   PERM, NRHS, IPARM, MSGlvl, B, X, ERROR, DPARM)

INTEGER*4 PT(64)
INTEGER*4 MAXFCT, MNUM, MTYPE, PHASE, N, NRHS, ERROR,
           IA(*), JA(*), PERM(*), IPARM(*)

REAL*8 DPARM(64)
REAL*8 A(*), B(N,NRHS), X(N,NRHS)

On 64-bit architectures:

SUBROUTINE PARDISOINIT(PT, MTYPE, SOLVER, IPARM, DPARM, ERROR)
INTEGER*4 PT(64)
INTEGER*4 MTYPE, SOLVER, IPARM(64), DPARM, ERROR
REAL*8 DPARM(64)

```
SUBROUTINE PARDISO(PT, MAXFCT, MNUM, MTYPE, PHASE, N, A, IA, JA,
  PERM, NRHS, IPARM, MSGlvl, B, X, ERROR, DPARM)

INTEGER*8 PT(64)
INTEGER*4 MAXFCT, MNUM, MTYPE, PHASE, N, NRHS, ERROR,
   IA(*), JA(*), PERM(*), IPARM(*)
REAL*8 DPARM(64)
REAL*8 A(*), B(N,NRHS), X(N,NRHS)

The sparse direct solvers use 8-byte integer length internally that allows storage of factors with more
than $2^{32}$ nonzeros elements. The following are examples for calling Pardiso from a Fortran program,

```fortran
! Check license of the solver and initialize the solver
CALL PARDISOINIT(PT, MTYPE, SOLVER, IPARM, DPARM, ERROR)
! Solve matrix system
CALL PARDISO(PT, MAXFCT, MNUM, MTYPE, PHASE, N, A, IA, JA,
   PERM, NRHS, IPARM, MSGLVL, B, X, ERROR, DPARM)
```

and for calling PARDISO from a C program,

```c
/* Check license of the solver and initialize the solver */
pardisoinit(pt, &mtype, &solver, iparm, dparm, &error);

/* Solve matrix system */
pardiso(pt, &maxfct, &mnum, &mtype, &phase, &n, &a, &ia, &ja,
   &perm, &nrhs, &iparm, &msglvl, &b, &x, &error, &dparm)
```

### 2.1 Arguments of PARDISOINIT

PARDISOINIT checks the current license in the file `panua.lic`, located at the home directory, and initializes
the internal timer and the address pointer `PT`. It sets the solver default values according to the matrix
type.

**PT(64) — INTEGER**

*Input/Output*

*On entry:* This is the solver’s internal data address pointer. These addresses are passed to the
solver and all related internal memory management is organized through this pointer.

*On exit:* Internal address pointers.

**Note:** PT is a 32-bit or 64-bit integer array on 32 or 64 bit operating systems, respectively. It has
64 entries.

**Never** change PT!

**MTYPE — INTEGER**

*Input*

*On entry:* This scalar value defines the matrix type. Pardiso supports the following matrices

<table>
<thead>
<tr>
<th>TYPE</th>
<th>Matrix</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>real and structurally symmetric</td>
</tr>
<tr>
<td>2</td>
<td>real and symmetric positive definite</td>
</tr>
<tr>
<td>-2</td>
<td>real and symmetric indefinite</td>
</tr>
<tr>
<td>3</td>
<td>complex and structurally symmetric</td>
</tr>
<tr>
<td>4</td>
<td>complex and Hermitian positive definite</td>
</tr>
<tr>
<td>-4</td>
<td>complex and Hermitian indefinite</td>
</tr>
<tr>
<td>6</td>
<td>complex and symmetric</td>
</tr>
<tr>
<td>11</td>
<td>real and nonsymmetric</td>
</tr>
<tr>
<td>13</td>
<td>complex and nonsymmetric</td>
</tr>
</tbody>
</table>
**Note:** The parameter influences the pivoting method.

**SOLVER — INTEGER**

*On entry:* This scalar value defines the solver method that the user would like to use.

<table>
<thead>
<tr>
<th>SOLVER</th>
<th>Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>sparse direct solver</td>
</tr>
<tr>
<td>1</td>
<td>multi-recursive iterative solver</td>
</tr>
</tbody>
</table>

**IPARM(64) — INTEGER**

*On entry:* IPARM is an integer array of size 64 that is used to pass various parameters to Pardiso and to return some useful information after the execution of the solver. PARDISOINIT fills IPARM(1), IPARM(2), and IPARM(4) through IPARM(64) with default values and uses them.

See section 2.3 for a detailed description.

**Note:** There is no default value for IPARM(3), which reflects the number of processors and this value must always be supplied by the user.

**DPARM(64) — REAL**

*On entry:* DPARM is a double-precision array of size 64 that is used to pass various parameters to Pardiso and to return some useful information after the execution of the solver. The array will only be used to initialize the multi-recursive iterative linear solver in Pardiso. PARDISOINIT fills DPARM(1) through DPARM(64) with default values and uses them.

See section 2.4 for a detailed description.

**ERROR — INTEGER**

*On output:* The error indicator

<table>
<thead>
<tr>
<th>Error</th>
<th>Information</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>No error.</td>
</tr>
<tr>
<td>-10</td>
<td>No license file pardiso.lic found</td>
</tr>
<tr>
<td>-11</td>
<td>License is expired</td>
</tr>
<tr>
<td>-12</td>
<td>Wrong username or hostname</td>
</tr>
</tbody>
</table>

### 2.2 Arguments of Pardiso

Solving a linear system is split into four tasks: (1) analysis and symbolic factorization, (2) numerical factorization, (3) forward and backward substitution including iterative refinement, and finally, (4) termination to release all internal solver memory. The calling sequence and description of the Pardiso parameters is as follows: when an input data structure is not accessed in a call, a NULL pointer or any valid address can be passed as a place holder for that argument.

**PT(64) — INTEGER**

*see PARDISOINIT.*

**MAXFCT — INTEGER**

*On entry:* Maximal number of factors with identical nonzero sparsity structure that the user would like to keep at the same time in memory. It is possible to store several different factorizations with the same nonzero structure at the same time in the internal data management of the solver. In many applications this value is equal to 1.
**Note:** Matrices with different sparsity structure can be kept in memory with different memory address pointers PT.

**MNUM — INTEGER**

*Input*

*On entry:* Actual matrix for the solution phase. With this scalar the user can define the matrix that he would like to factorize. The value must be: \(1 \leq \text{MNUM} \leq \text{MAXFCT} \). In many applications this value is equal to 1.

**MTYPE — INTEGER**

*Input*

see **PARDISOINIT**. **Note:** The parameter influences the pivoting method.

**PHASE — INTEGER**

*Input*

*On entry:* **PHASE** controls the execution of the solver. It is a two-digit integer \(ij\) (\(10i + j, 1 \leq i \leq 3, i \leq j \leq 3\) for normal execution modes). The \(i\) digit indicates the starting phase of execution, and \(j\) indicates the ending phase. Pardiso has the following phases of execution:

1. Phase 1: Fill-reduction analysis and symbolic factorization
2. Phase 2: Numerical factorization
3. Phase 3: Forward and Backward solve including iterative refinement
4. Termination and Memory Release Phase (\(\text{PHASE} \leq 0\))

If a previous call to the routine has computed information from previous phases, execution may start at any phase:

<table>
<thead>
<tr>
<th>PHASE</th>
<th>Solver Execution Steps</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>Analysis</td>
</tr>
<tr>
<td>12</td>
<td>Analysis, numerical factorization</td>
</tr>
<tr>
<td>13</td>
<td>Analysis, numerical factorization, solve, iterative refinement</td>
</tr>
<tr>
<td>22</td>
<td>Numerical factorization</td>
</tr>
<tr>
<td>-22</td>
<td>Selected Inversion</td>
</tr>
<tr>
<td>23</td>
<td>Numerical factorization, solve, iterative refinement</td>
</tr>
<tr>
<td>33</td>
<td>Solve, iterative refinement</td>
</tr>
<tr>
<td>0</td>
<td>Release internal memory for (L) and (U) matrix number (\text{MNUM})</td>
</tr>
<tr>
<td>-1</td>
<td>Release all internal memory for all matrices</td>
</tr>
</tbody>
</table>

**Note:** The analysis phase needs the numerical values of the matrix \(A\) in case of scalings (IPARM(11)=1) and symmetric weighted matching (IPARM(13)=1 (normal matching) or IPARM(13)=2 (advanced matchings)).

**N — INTEGER**

*Input*

*On entry:* Number of equations in the sparse linear systems of equations \(A \times X = B\).

*Constraint:* \(N > 0\).

**A(*) — REAL/COMPLEX**

*Input*

*On entry:* Nonzero values of the coefficient matrix A. The array A contains the values corresponding to the indices in JA. The size and order of A is the same as that of JA. The coefficients can be either real or complex. The matrix must be stored in compressed sparse row format with increasing values of JA for each row. Refer to Figure 2 for more details.

**Note:** For symmetric or structural symmetric matrices it is important that the diagonal elements are also available and stored in the matrix. If the matrix is symmetric or structural symmetric, then the array A is only accessed in the factorization phase, in the triangular solution with iterative refinement. Nonsymmetric matrices are accessed in all phases of the solution process.
**IA(N+1)** — INTEGER

*Input*

*On entry:* IA is an integer array of size N+1. IA(i) points to the first column index of row i in the array JA in compressed sparse row format. Refer to Figure 2 for more details.

**Note:** The array is accessed in all phases of the solution process. Note that the row and column numbers start from 1.

**JA(*)** — INTEGER

*Input*

*On entry:* The integer array JA contains the column indices of the sparse matrix A stored in compress sparse row format. The indices in each row must be sorted in increasing order.

**Note:** The array is accessed in all phases of the solution process. For symmetric and structurally symmetric matrices it is assumed that zero diagonal elements are also stored in the list of nonzeros in A and JA. For symmetric matrices the solver needs only the upper triangular part of the system as shown in Figure 2.

**PERM(N)** — INTEGER

*Input*

*On entry:* The user can supply his own fill-in reducing ordering to the solver. This permutation vector is only accessed if **IPARM(5) = 1**.

**Note:** The permutation PERM must be a vector of size N. Let A be the original matrix and $B = P A P^T$ be the permuted matrix. The array **PERM** is defined as follows. Row (column) of A is the **PERM(i)** row (column) of B. The numbering of the array must start by 1 and it must describe a permutation.

**NRHS** — INTEGER

*Input*

*On entry:* NRHS is the number of right-hand sides that need to be solved for.

**IPARM(64)** — INTEGER

*Input/Output*

*On entry:* **IPARM** is an integer array of size 64 that is used to pass various parameters to Pardiso and to return some useful information after the execution of the solver. If **IPARM(1)** is 0, then **PARDISOINIT** fills **IPARM(2) and IPARM(4) through IPARM(64)** with default values.

See section 2.3 for a detailed description.

*On output.* Some **IPARM** values will contain useful information, e.g. numbers of nonzeros in the factors, etc.

**Note:** There is no default value for **IPARM(3)** and this value must always be supplied by the user, regardless of whether **IPARM(1)** is 0 or 1.

**MSGLVL** — INTEGER

*Input*

*On entry:* Message level information. If **MSGLVL=0** then Pardiso generates no output. If **MSGLVL=1** the direct solver prints statistical information to the screen. The multi-recursive iterative solver prints statistical information in the file pardiso-ml.out.

**B(N,NRHS)** — REAL/COMPLEX

*Input/Output*

*On entry:* Right-hand side vector/matrix.

*On output:* The array is replaced by the solution if **IPARM(6)=1**.

**Note:** B is only accessed in the solution phase.

**X(N,NRHS)** — REAL/COMPLEX

*Output*

*On output:* Solution if **IPARM(6)=0**.

**Note:** X is only accessed in the solution phase.

**ERROR** — INTEGER

*Output*

*On output:* The error indicator
**ERROR** | Information
---|---
0 | No error.
-1 | Input inconsistent.
-2 | Not enough memory.
-3 | Reordering problem.
-4 | Zero pivot, numerical fact. or iterative refinement problem.
-5 | Unclassified (internal) error.
-6 | Preordering failed (matrix types 11, 13 only).
-7 | Diagonal matrix problem.
-8 | 32-bit integer overflow problem.
-10 | No license file pardiso.lic found.
-11 | License is expired.
-12 | Wrong username or hostname.
-100 | Reached maximum number of Krylov-subspace iteration in iterative solver.
-101 | No sufficient convergence in Krylov-subspace iteration within 25 iterations.
-102 | Error in Krylov-subspace iteration.
-103 | Break-Down in Krylov-subspace iteration.

**DPARM(64)** — REAL

On entry: DPARM is a double-precision array of size 64 that is used to pass various parameters to Pardiso and to return some useful information after the execution of the iterative solver. The array will be used in the multi-recursive iterative linear solver in Pardiso. This iterative solver is used in case of IPARM(32)=1. In addition, DPARM returns double return values from the direct solver such as the determinant for symmetric indefinite matrices or unsymmetric matrices. See section 2.4 for a detailed description.

### 2.3 IPARM Control Parameters

All parameters are integer values.

**IPARM(1)** — **Use default values.**

On entry: If IPARM(1)=0, then IPARM(2) and IPARM(4) through IPARM(64) are filled with default values, otherwise the user has to supply all values in IPARM.

Note: If IPARM(1) ≠ 0 on entry, the user has to supply all values form IPARM(2) to IPARM(64).

**IPARM(2)** — **Fill-In reduction reordering.**

On entry: IPARM(2) controls the fill-in reducing ordering for the input matrix. If IPARM(2) is 0 then the minimum degree algorithm is applied [6], if IPARM(2) is 2, the solver uses the nested dissection algorithm from the METIS package version 4.0.1 [2]. If IPARM(2) is 3, the solver uses the nested dissection algorithm from the METIS package version 5.1. If IPARM(2) is 4, the solver uses the approximate minimum degree algorithm from the AMD package.

The default value of IPARM(2) is 2.

**IPARM(3)** — **Number of processors.**

On entry: IPARM(3) must contain the number of processors that are available for parallel execution. The number must be equal to the OpenMP environment variable OMP_NUM_THREADS.

Note: If the user has not explicitly set OMP_NUM_THREADS, then this value can be set by the operating system to the maximal numbers of processors on the system. It is therefore always recommended to control the parallel execution of the solver by explicitly setting OMP_NUM_THREADS. If fewer processors are available than specified, the execution may slow down instead of speeding up.
There is no default value for IPARM(3).

**IPARM(4) — Preconditioned CGS.**  
*Input*

*On entry:* This parameter controls preconditioned CGS [16] for nonsymmetric or structural symmetric matrices and Conjugate-Gradients for symmetric matrices. IPARM(4) has the form

\[
\text{IPARM}(4) = 10 \cdot L + K \geq 0.
\]

The values \( K \) and \( L \) have the following meaning:

<table>
<thead>
<tr>
<th>Value ( K )</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>K can be chosen independently of MTYPE.</td>
<td></td>
</tr>
<tr>
<td>LU was computed at least once before call with IPARM(4) (!= 0).</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>LU preconditioned CGS iteration for ( A \times X = B ) is executed.</td>
</tr>
<tr>
<td>2</td>
<td>LU preconditioned CG iteration for ( A \times X = B ) is executed.</td>
</tr>
</tbody>
</table>

**Value L:** The value \( L \) controls the stopping criterion of the Krylov-Subspace iteration:

\[
\epsilon_{\text{CGS}} = 10^{-L} \text{ is used in the stopping criterion } \|dx_i\|/\|dx_1\| < \epsilon_{\text{CGS}} \text{ with } \|dx_i\| = \| (LU)^{-1} r_i \| \text{ and } r_i \text{ is the residual at iteration } i \text{ of the preconditioned Krylov-Subspace iteration.}
\]

**Strategy:** A maximum number of 150 iterations is set on the expectation that the iteration will converge before consuming half the factorization time. Intermediate convergence rates and the norm of the residual are checked and can terminate the iteration process. The basic assumption is: LU was computed at least once — hence it can be recomputed if the iteration process converges slowly.

If PHASE=23: the factorization for the supplied \( A \) is automatically recomputed in cases where the Krylov-Subspace iteration fails and the direct solution is returned. Otherwise the solution from the preconditioned Krylov Subspace iteration is returned.

If PHASE=33: the iteration will either result in success or an error (ERROR=4), but never in a recomputation of LU. A simple consequence is: (A,IA,JA) may differ from the (A,IA,JA)’ supplied to compute LU (MTYPE, N have to be identical). More information on the failure can be obtained from IPARM(20).

**Note:** The default is IPARM(4)=0 and other values are only recommended for advanced users.

**Examples:**

<table>
<thead>
<tr>
<th>IPARM(4)</th>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>31</td>
<td>LU-preconditioned CGS iteration with a stopping tolerance of ( 10^{-3} ) for nonsymmetric matrices</td>
</tr>
<tr>
<td>61</td>
<td>LU-preconditioned CGS iteration with a stopping tolerance of ( 10^{-6} ) for nonsymmetric matrices</td>
</tr>
<tr>
<td>62</td>
<td>LU-preconditioned CG iteration with a stopping tolerance of ( 10^{-6} ) for symmetric matrices</td>
</tr>
</tbody>
</table>

**IPARM(5) — User permutation.**  
*Input*

*On entry:* IPARM(5) controls whether a user-supplied fill-in reducing permutation is used instead of the integrated multiple-minimum degree or nested dissection algorithm.

**Note:** This option may be useful for testing reordering algorithms or adapting the code to special application problems (e.g. to move zero diagonal elements to the end of \( PAP^T \)). The permutation must be a vector of size \( N \). Let \( A \) be the original matrix and \( B = PAP^T \) be the permuted matrix. The array PERM is defined as follows. Row (column) of \( A \) is the PERM(i) row (column) of \( B \). The numbering of the array must start at 1.

The default value of IPARM(5) is 0.
IPARM(6) — Write solution on X.

On entry: If IPARM(6) is 0, which is the default, then the array X contains the solution and the value of B is not changed. If IPARM(6) is 1 then the solver will store the solution on the right-hand side B.

Note: The array X is always changed.
The default value of IPARM(6) is 0.

IPARM(7) — Number of performed iterative refinement steps.

On output: The number of iterative refinement steps performed during the solve step are reported in IPARM(7).

IPARM(8) — Iterative refinement steps.

On entry: On entry to the solve and iterative refinement step, IPARM(8) should be set to the maximum number of iterative refinement steps that the solver will perform. Iterative refinement will stop if a satisfactory level of accuracy of the solution in terms of backward error has been achieved.
The solver will automatically perform two steps of iterative refinement when perturbed pivots have occurred during the numerical factorization and IPARM(8) was equal to zero.
The number of iterative refinement steps is reported in IPARM(7).

Note: If IPARM(8)<0 the accumulation of the residual is using complex*32 or real*16 in case that the compiler supports it (GNU gfortran does not). Perturbed pivots result in iterative refinement (independent of IPARM(8)=0) and the iteration number executed is reported in IPARM(7).
If IPARM(8)=-99 then the automatic iterative refinement steps are turned off.
The default value is 0.

IPARM(9) — Unused.

This value is reserved for future use. Value must be set to 0.

IPARM(10) — Pivot perturbation.

On entry: IPARM(10) instructs Pardiso how to handle small pivots or zero pivots for nonsymmetric matrices (MTYPE=11, MTYPE=13) and symmetric matrices (MTYPE=-2, MTYPE=-4, or MTYPE=6). For these matrices the solver uses a complete supernode pivoting approach. When the factorization algorithm reaches a point where it cannot factorize the supernodes with this pivoting strategy, it uses a pivoting perturbation strategy similar to [5, 13], compare 2.6. The magnitude of the potential pivot is tested against a constant threshold of \( \alpha = \epsilon \cdot ||A_2||_\infty \), where \( \epsilon = 10^{-\text{IPARM(10)}} \) and \( ||A_2||_\infty \) is the \( \infty \)-norm of the scaled and permuted matrix A. Any tiny pivots \( l_{ii} \) encountered during elimination are set to the sign(\( l_{ii} \)) \( \cdot \epsilon \cdot ||A_2||_\infty \).
The default value of IPARM(10) is 13 and therefore \( \epsilon = 10^{-13} \) for nonsymmetric matrices. The default value of IPARM(10) is 8 and \( \epsilon = 10^{-8} \) for symmetric indefinite matrices (MTYPE=-2, MTYPE=-4, or MTYPE=6).

IPARM(11) — Scaling vectors.

On entry: Pardiso can use a maximum weighted matching algorithm to permute large elements close the diagonal and to scale the matrix so that the largest elements are equal to 1 and the absolute value of the off-diagonal entries are less or equal to 1. This scaling method is applied to nonsymmetric matrices (MTYPE=11 or MTYPE=13). The scaling can also be used for symmetric indefinite matrices (MTYPE=-2, MTYPE=-4, or MTYPE=6) if symmetric weighted matchings is applied (IPARM(13)=1 or IPARM(13)=3).

Note: It is recommended to use IPARM(11)=1 (scaling) and IPARM(13)=1 (matchings) for highly indefinite symmetric matrices e.g. from interior point optimizations or saddle point problems. It is also very important to note that the user must provide in the analysis phase (PHASE=11) the numerical values of the matrix A if IPARM(11)=1 (scaling) or IPARM(13)=1 or 2 (matchings).
The default value of $\text{IPARM}(11)$ is 1 for nonsymmetric matrices ($\text{MTYPE}=11, \text{MTYPE}=13$) and $\text{IPARM}(11)$ is 0 for symmetric matrices ($\text{MTYPE}=-2, \text{MTYPE}=-4, \text{MTYPE}=6$).

**IPARM(12) — Solving with transpose matrix.** Input

On entry: Solve a system $A^T x = b$ by using the factorization of $A$.

The default value of $\text{IPARM}(12)$ is 0. $\text{IPARM}(12)=1$ indicates that you would like to solve transposed system $A^T x = b$.

**IPARM(13) — Improved accuracy using (non-)symmetric weighted matchings.** Input

On entry: Pardiso can use a maximum weighted matching algorithm to permute large elements close the diagonal. This strategy adds an additional level of reliability to our factorization methods and can be seen as a complement to the alternative idea of using more extensive pivoting techniques during the numerical factorization.

Note: It is recommended to use $\text{IPARM}(11)=1$ (scalings) and $\text{IPARM}(13)=1$ (normal matchings) and $\text{IPARM}(13) = 2$ (advanced matchings, higher accuracy) for highly indefinite symmetric matrices e.g. from interior point optimizations or saddle point problems. Note that the user must provide in the analysis phase ($\text{PHASE}=11$) the numerical values of the matrix $A$ in both of these cases.

The default value of $\text{IPARM}(13)$ is 1 for nonsymmetric matrices ($\text{MTYPE}=11$ or $\text{MTYPE}=13$) and 0 for symmetric matrices ($\text{MTYPE}=-2, \text{MTYPE}=-4, \text{MTYPE}=6$).

**IPARM(14) — Number of perturbed pivots.** Output

On output: After factorization, $\text{IPARM}(14)$ contains the number of perturbed pivots during the elimination process for $\text{MTYPE}=-2, -4, 6, 11, 13$.

**IPARM(15) — Peak memory symbolic factorization.** Output

On output: $\text{IPARM}(15)$ provides the user with the total peak memory in KBytes that the solver needed during the analysis and symbolic factorization phase. This value is only computed in phase 1.

**IPARM(16) — Permanent memory symbolic factorization.** Output

On output: $\text{IPARM}(16)$ provides the user with the permanently needed memory — hence allocated at the end of phase 1 — in KBytes to execute the factorization and solution phases. This value is only computed in phase 1 and $\text{IPARM}(15) > \text{IPARM}(16)$ holds.

**IPARM(17) — Memory numerical factorization and solution.** Output

On output: $\text{IPARM}(17)$ provides the user with the total double precision memory consumption (KBytes) of the solver for the factorization and solution phases. This value is only computed in phase 2.

Note: Total peak memory consumption is

\[
\text{peakmem} = \max (\text{IPARM}(15), \text{IPARM}(16)+\text{IPARM}(17)).
\]

**IPARM(18) — Number nonzeros in factors.** Input/Output

On entry: if $\text{IPARM}(18) < 0$ the solver will report the number of nonzeros in the factors.

On output: The numbers of nonzeros in the factors are returned to the user.

The default value of $\text{IPARM}(18)$ is -1.

**IPARM(19) — GFlops of factorization.** Input/Output

On entry: if $\text{IPARM}(19)<0$ the solver will compute the number of GFlops ($10^9$) to factor the matrix $A$. This will increase the reordering time.

On output: Number of GFlops ($10^9$ operations) needed to factor the matrix $A$. 

The default value of IPARM(19) is 0.

**IPARM(20) — CG/CGS diagnostics.**

*Output*

On output: The value is used to give CG/CGS diagnostics (e.g. the number of iterations and cause of failure):

- IPARM(20)> 0: CGS succeeded and the number of iterations is reported in IPARM(20).
- IPARM(20)< 0: Iterations executed, but CG/CGS failed. The error details in IPARM(20) are of the form:

\[
\text{IPARM}(20)< 0 = -\text{it}_c\text{gs} \times 10^{-\text{cgs\_error}}.
\]

If PHASE was 23, then the factors L, U have been recomputed for the matrix A and the error flag ERROR should be zero in case of a successful factorization. If PHASE was 33, then ERROR = -4 will signal the failure.

**Description of cgs\_error:**

<table>
<thead>
<tr>
<th>cgs_error</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Too large fluctuations of the residual;</td>
</tr>
<tr>
<td>2</td>
<td>[</td>
</tr>
<tr>
<td>3</td>
<td>Stopping criterion not reached at max_it_cgs;</td>
</tr>
<tr>
<td>4</td>
<td>Perturbed pivots caused iterative refinement;</td>
</tr>
<tr>
<td>5</td>
<td>Factorization is too fast for this matrix. It is better to use the factorization method with IPARM(4) = 0.</td>
</tr>
</tbody>
</table>

**IPARM(21) — Pivoting for symmetric indefinite matrices.**

*Input*

On entry: IPARM(21) controls the pivoting method for sparse the matrices. If IPARM(21) is 0, then 1×1 diagonal pivoting is used. If IPARM(21) is 1, then 1×1 and 2×2 Bunch-Kaufman pivoting will be used within the factorization process. If IPARM(21) is 2, then 1×1 diagonal pivoting and a new rescaling is used which is recomputed during the factorization process. If IPARM(21) is 3, then 1×1 and 2×2 Bunch-Kaufman pivoting and a new rescaling is used which is recomputed during the factorization process.

**Note:** It is also recommended to use IPARM(11)=1 (scaling) and IPARM(13)=1 or 2 (matchings) for highly indefinite symmetric matrices e.g. from interior point optimizations or saddle point problems.

Bunch-Kaufman pivoting is available for matrices MTYPE=-2, -4, 6.

The default value of IPARM(21) is 1, but we recommend also IPARM(21)=3.

**IPARM(22) — Inertia: Number of positive eigenvalues.**

*Output*

On output: The number of positive eigenvalues for symmetric indefinite matrices is reported in IPARM(22).

**IPARM(23) — Inertia: Number of negative eigenvalues.**

*Output*

On output: The number of negative eigenvalues for symmetric indefinite matrices is reported in IPARM(23).

**IPARM(24) — Parallel Numerical Factorization.**

*Input*

On entry: This parameter selects the scheduling method for the parallel numerical factorization. If IPARM(24)=0 then the solver will use a parallel factorization that has been used in the solver during the last years. IPARM(24)=1 selects a two-level scheduling algorithms which should result in a better parallel efficiency.

The default value of IPARM(24) is 1.
**IPARM(25) — Parallel Forward/Backward Solve.** Input

*On entry:* The parameter controls the parallelization for the forward and backward solve. 
*IPARM(25)=0* indicates that a sequential forward and backward solve is used, whereas 
*IPARM(25)=1* selects a parallel solve.

The default value of *IPARM(25)* is 1.

**IPARM(26) — Splitting of Forward/Backward Solve.** Input

*On entry:* The user can apply a partial forward and backward substitution with this parameter. 
*IPARM(26)=0* indicates that a normal solve step is performed with both the factors $L$ and $U$. With unsymmetric matrix *IPARM(26)=1* indicates that a forward solve step is performed with the factors $L$ or $U^T$. 
*IPARM(26)=2* indicates that a backward solve step is performed with the factors $U$ or $L^T$ — compare *IPARM(12)*. With symmetric matrix *IPARM(26)=1* or -1 indicates that a forward solve step is performed with the factors $L$ or $U^T$. 
*IPARM(26)=-2* indicates that a backward solve step is performed with the factor $D$. 
*IPARM(26)=-3* indicates that a backward solve step is performed with the factors $U$ or $L^T$. 
*IPARM(26)=2* indicates that a backward solve step is performed with the factors $D$ and $U$ or $L^T$ — compare *IPARM(12)*.

The default value of *IPARM(26)* is 0.

**IPARM(27) — Recomputation of Scaling.** Input

*On entry:* The parameter controls the recomputation of the scaling during the numerical factorization. 
*IPARM(27)=1* indicates that a scaling is recomputed, 
*IPARM(27)=0* indicates the scaling is used from a previous symbolic reordering. Noted that this option is only active if *IPARM(13)>0* (matchings).

The default value of *IPARM(13)* is 0.

**IPARM(28) — Parallel Reordering for METIS.** Input

*On entry:* The parameter controls the parallel execution of the METIS reordering. 
*IPARM(28)=1* indicates that a parallel reordering is performed, 
*IPARM(28)=0* indicates a sequential reordering.

The default value of *IPARM(28)* is 0.

**IPARM(29) — Switch between 32-bit and 64-bit factorization.** Input

*On entry:* The parameter controls the IEEE accuracy of the solver 
*IPARM(29)=0* indicates that a 64-bit factorization is performed and 
*IPARM(29)=1* indicates a 32-bit factorization.

The default value of *IPARM(29)* is 0. The option is only available for symmetric indefinite matrices (*MTYPE*=-2) and for real unsymmetric matrices (*MTYPE*=-11).

**IPARM(30) — Control the size of the supernodes.** Input

*On entry:* The parameter controls the column size of the supernodes, e.g., 
*IPARM(30)=40* indicates that 40 columns are used for each supernode.

The default value of *IPARM(30)* is 80 for symmetric matrices.

**IPARM(31) — Partial solve for sparse right-hand sides and sparse solution.** Input

*On entry:* The parameter controls the solution method in case that the user is interested in only a few components of the solution vector $X$. The user has to define the vector components in the input vector *PERM*. An entry *PERM(i)=1* means that either the right-hand side is nonzero at this
component, and it also defines that the solution $i$-th component should be computed in the vector $X$. The vector $\text{PERM}$ must be present in all phases of Pardiso.

The default value of $\text{IPARM}(31)$ is 0. An entry equal $\text{IPARM}(31)=1$ selects the sparse right-hand sides solution method.

**$\text{IPARM}(32)$ — Use the multi-recursive iterative linear solver.**

*Input*

*On entry:* The default value of $\text{IPARM}(32)$ is 0 (sparse direct solver) and $\text{IPARM}(32)=1$ will use the iterative method in Pardiso.

**$\text{IPARM}(33)$ — Determinant of a matrix.**

*Input*

*On entry:* $\text{IPARM}(33)=1$ will compute the determinant of matrices and will return in $\text{DPARM}(33)$ the real part of the determinant and, if necessary, in $\text{DPARM}(32)$ the complex part of the determinant. *On output:* The parameter returns the natural logarithm of the determinant of a sparse matrix $A$.

The default value of $\text{IPARM}(33)$ is 0.

**$\text{IPARM}(34)$ — Identical solution independent on the number of processors.**

*Input*

*On entry:* $\text{IPARM}(34)=1$ will always compute the identical solution independent on the number of processors that Pardiso is using during the parallel factorization and solution phase. This option is only working with the METIS reordering ($\text{IPARM}(2)=2$), and you also have to select a parallel two-level factorization ($\text{IPARM}(24)=1$) and a parallel two-level solution phase ($\text{IPARM}(25)=1$). In addition, the user has define the upper limit of processors in $\text{IPARM}(3)$ for which he would like to have the same results.

*Example* $\text{IPARM}(3)=8$, $\text{IPARM}(2)=2$, and $\text{IPARM}(34)=\text{IPARM}(24)=\text{IPARM}(25)=1$: This option means that the user would like to have identical results for up to eight processors. The number of cores that the user is actually using in Pardiso can be set by the OpenMP environment variable $\text{OMP}_\text{NUM}\_\text{THREADS}$. In our example, the solution is always the same for $\text{OMP}_\text{NUM}\_\text{THREADS}=1$ up to $\text{OMP}_\text{NUM}\_\text{THREADS}=\text{IPARM}(3)=8$.

**$\text{IPARM}(35)$ — Unused.**

*Output*

This value is reserved for future use. Value MUST be set to 0.

**$\text{IPARM}(36)$ — Selected inversion for $A^{-1}_{ij}$.**

*Input*

*On entry:* $\text{IPARM}(36)=1$ will control the selected inversion process based on the internal $L$ and $U$ factors. If $\text{IPARM}(36)=0$ and $\text{PHASE}=-22$, Pardiso will overwrite these factors with the selected inverse elements of $A^{-1}$. If $\text{IPARM}(36)=1$ and $\text{PHASE}=-22$, Pardiso will not overwrite these factors. It will instead allocate additional memory of size of the numbers of elements in $L$ and $U$ to store these inverse elements.

The default value of $\text{IPARM}(36)$ is 0.

The following code fragment shows how to use the selected inversion process in Pardiso from a Fortran program:

```
! Reorder and factor a matrix system
  PHASE=12
  CALL PARDISO(PT, MAXFCT, MNUM, MTYPE, PHASE,N, A, IA, JA, PERM, NRHS, IPARM, MSG_LVL, B, X, ERROR, DPARAM)

! Compute selected inversion process and return inverse elements in CSR structure (a,ia,ja)
```
IPARM(36) — 0
CALL PARDISO(PT, MAXFCT, MNUM, MTYPE, PHASE, N, A, IA, JA,
PERM, NRHS, IPARM, MSGlvl, B, X, ERROR, DPARM)

IPARM(37) — Selected inversion for $A^{-1}_{ij}$.

On entry: IPARM(37) will control the selected inversion process for symmetric matrices. If IPARM(37)=0, Pardiso will return the selected inverse elements $A^{-1}_{ij}$ in upper symmetric triangular CSR format. If IPARM(37)=1, Pardiso will return the selected inverse elements $A^{-1}_{ij}$ in full symmetric triangular CSR format.

The default value of IPARM(37) is 0.

IPARM(38) — Schur-complement computation.

On entry: IPARM(38) will control the Schur-complement computation and it will compute the partial Schur complement $S = A_{22} - A_{21}A^{-1}_{11}A_{12}$ for the matrix $A = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}$. IPARM(38) indicates the numbers of rows/columns size of the matrix $A_{22}$.

The default value of IPARM(38) is 0.

The following code fragment shows how to use the Schur-complement method in Pardiso from a C++ program to compute a Schur-complement of size 5.

```
// Reorder and factor a matrix system
int phase = 12;
int nrows = 5;
iparm[38-1] = nrows;
phase = 12;
pardiso(pt, &maxfct, &mnum, &mtype, &phase, &n, a, ia, ja, &idum,
&nb, &iparm[0], &msglvl, &ddum, &ddum, &error, dparm);
```

IPARM(39) — Nonzeros in Schur-complement $S = A_{22} - A_{21}A^{-1}_{11}A_{12}$.

On Output: The numbers of nonzeros in the Schur-complement $S = A_{22} - A_{21}A^{-1}_{11}A_{12}$.

IPARM(40) — Permutation.

On Output: The internal fill-in reducing permutation is returned to the user. If IPARM(40)=1, PARDISO will return the internal permutation to the user.

The default value of IPARM(40) is 0.

IPARM(41) — Incremental Update.

On Input: Please contact Olaf Schenk in case of questions or examples. The default value of IPARM(41) is 0.

IPARM(42) to IPARM(49) — Unused.

These values are reserved for future use. Value MUST be set to 0.
**IPARM(50)** — *Use of out-of-core solver option.*

*Input*

*On Input:* \( \text{IPARM}(50) = 0 \), then PARDISO will always use the in-core version. \( \text{IPARM}(50) \) defines the number of submatrices that are written to files with names \( \text{pardiso-factor-diag.mat} \), \( \text{small-ooc-domain} \), \( \text{pardiso-ooc-domain} \), and \( \text{pardiso-sep-domain} \). The number of files is used as \( 2^{\text{IPARM}(50)} \) in the solver. This option is only available for real symmetric indefinite matrices (\( \text{MTYPE} = -2 \)) and complex symmetric matrices (\( \text{MTYPE} = 6 \)).

The default value of \( \text{IPARM}(50) \) is 0.

**IPARM(51)** to **IPARM(52)** — *Unused.*

*Output*

These values are reserved for future use. Value MUST be set to 0.

**IPARM(53)** — *Block size.*

*Input*

*On Input:* This parameter sets the block size of block matrices.

- \( \text{IPARM}(53) = 0 \): Normal CSR matrix format is expected on input, block compression is not used
- \( \text{IPARM}(53) \geq 1 \): Block CSR matrix format is expected on input, \( \text{IPARM}(53) \) is the block size, block compression is used

The default value of \( \text{IPARM}(53) \) is 0.

**IPARM(54)** — *Dense columns.*

*Input*

*On Input:* This parameter sets the number of dense columns at the end of the matrix.

- \( \text{IPARM}(54) = 0 \): Dense column compression is not used
- \( \text{IPARM}(54) > 1 \): \( \text{IPARM}(54) \) dense columns are expected at the end of the matrix, dense column compression is used

The default value of \( \text{IPARM}(54) \) is 0.

**IPARM(55)** — *METIS 4.0.1 and METIS 5.1 ordering options.*

*Input*

*On Input:* If \( \text{IPARM}(55) = 0 \) then default values in METIS are used. If \( \text{IPARM}(55) = 1 \), then the next seven elements \( \text{IPARM}(56) \) to \( \text{IPARM}(62) \) are interpreted as follows.

The default value of \( \text{IPARM}(55) \) is 0. Pardiso uses a modified versions of METIS 4.0.1 and METIS 5.1 [2] and these versions are included in the libraries. The manual of METIS 4.0.1 is available at [here](#) and the user manual of METIS 5.1 is available [here](#). The non-default option is only available for the non-parallel version of METIS 4.0.1 (\( \text{IPARM}(28) = 0 \)) and for METIS 5.1.

**IPARM(56)** — *Matching type in METIS.*

*Input*

*On Input:* Determines the matching type. Possible values are:

- \( \text{IPARM}(56) = 1 \): Random Matching (RM).
- \( \text{IPARM}(56) = 2 \): Heavy-Edge Matching (HEM, only for METIS 4.0.1).
- \( \text{IPARM}(56) = 3 \): Sorted Heavy-Edge Matching (SHEM) (Default).

Experiments have shown that all three matching schemes perform quite well. In general SHEM is faster and RM is slower, but feel free to experiment with the other matching schemes.

The value of \( \text{IPARM}(56) \) will change option[1] in METIS 4.0.1 and METIS_OPTION_CTYPE in METIS 5.1

The default value of \( \text{IPARM}(56) \) is 3.

**IPARM(57)** — *Algorithm used during METIS initial partitioning.*

*Input*

*On Input:* Determines the algorithm used during initial partitioning. Possible values are:
- IPARM(57)=1: Edge-based region growing (Default).
- IPARM(57)=2: Node-based region growing.

The value of IPARM(57) will change option\,[2\] in METIS 4.0.1 and METIS\_OPTION\_PTYPE in METIS 5.1.

The default value of IPARM(57) is 1.

**IPARM(58) — Algorithm used for refinement.**

*Input*

On Input: Determines the algorithm used for refinement. Possible values are:

- IPARM(58)=1: Two-sided node FM refinement.
- IPARM(58)=2: One-sided node FM refinement (Default).

One-sided FM refinement is faster than two-sided, but in some cases two-sided refinement may produce better orderings. Feel free to experiment with this option.

The value of IPARM(58) will change option[3] in METIS 4.0.1 and METIS\_OPTION\_RTYPE in METIS 5.1.

The default value of IPARM(58) is 2.

**IPARM(59) — Used for METIS debugging purposes.**

*Input*

On Input: Used for debugging purposes.

- IPARM(59) = 0: no output.
- IPARM(59) > 0: output from METIS (statistics, etc, please refer to the METIS manual).

The value of IPARM(59) will change option[4] in METIS 4.0.1 and METIS\_OPTION\_DBGLVL in METIS 5.1.

The default value of IPARM(59) is 0.

**IPARM(60) — Compression of the graph in METIS.**

*Input*

On Input: Used to select whether or not to compress the graph and to order connected components separately. The possible values and their meaning are as follows:

- IPARM(60)=0: Do not try to compress the graph and do not order each connected component separately.
- IPARM(60)=1: Try to compress the graph. (A compressed graph is actually formed if the size of the graph can be reduced by at least 15%).
- IPARM(60)=2: Order each connected component of the graph separately. This option is particularly useful when after a few levels of nested dissection, the graph breaks up in many smaller disconnected subgraphs. This is true for certain types of LP matrices.
- IPARM(60)=3: Try to compress the graph and also order each connected component separately.

The value of IPARM(60) will change option[5] in METIS 4.0.1 and METIS\_OPTION\_COMPRESS in METIS 5.1.

The default value of IPARM(60) is 1 (METIS 4.0.1) and IPARM(60) is 0 (METIS 5.1).

**IPARM(61) — Remove any vertices with high degree in METIS.**

*Input*

On Input: Used to control whether or not the ordering algorithm should remove any vertices with high degree (i.e., dense columns). This is particularly helpful for certain classes of LP matrices, in which there is a few vertices that are connected to many other vertices. By removing these vertices prior to ordering, the quality and the amount of time required to do the ordering improves. The possible values are as follows:

- IPARM(61) = 0: Do not remove any vertices
• **IPARM(61) = x:** Where \( x > 0 \), instructs the algorithm to remove any vertices whose degree is greater than \( 0.1 \times x \times \) (average degree). For example if \( x = 40 \), and the average degree is 5, then the algorithm will remove all vertices with degree greater than 20. The vertices that are removed are ordered last (i.e., they are automatically placed in the top-level separator).

Good values are often in the range of 60 to 200 (i.e., 6 to 20 times more than the average).

The value of **IPARM(61)** will change option[6] in METIS 4.0.1 and METIS_OPTION_PFACTOR in METIS 5.1.

The default value of **IPARM(61)** is 200.

**IPARM(62) — Trial number of separators in nested dissection.**

Input

*On Input:* Used to determine how many separators to find at each step of nested dissection. The larger the number of separators found at each step, the higher the runtime and better the quality is (in general). The default value is 1, unless the graph has been compressed by more than a factor of 2, in which case it becomes 2. Reasonable values are in the range of 1 to 5. For most problems, a value of 5 increases the runtime by a factor of 3.

- **IPARM(62) = 1:** only one trial.
- **IPARM(62) = k:** \( k \) trials.

The value of **IPARM(62)** will change option[7] in METIS 4.0.1 and METIS_OPTION_NSEPS in METIS 5.1.

The default value of **IPARM(62)** is 1.

**IPARM(63) to IPARM(65) — Unused.**

Output

These values are reserved for future use. Value MUST be set to 0.

### 2.4 DPARM Control Parameters

All parameters are 64-bit double values. These parameters are only important for the multirecursive iterative solver (except **DPARM(33)** which returns the determinant of symmetric real indefinite matrices).

A log-file with detailed information of the convergence process is written in pardiso-ml.out. The initial guess for the solution has to be provided in the solution vector \( x \). A vector \( x \) initialized with zeros is recommended.

**DPARM(1) — Maximum number of Krylov-Subspace Iteration.**

*Input*

*On entry:* This values sets the maximum number of SQMR iteration for the iterative solver.

The default value of **DPARM(1)** is 300.

**DPARM(2) — Relative Residual Reduction.**

*Input*

*On entry:* This value sets the residual reduction for the SQMR Krylov-Subspace Iteration.

The default value of **DPARM(2)** is 1e-6.

**DPARM(3) — Coarse Grid Matrix Dimension.**

*Input*

*On entry:* This value sets the matrix size at which the multirecursive method switches to the sparse direct method.

The default value of **DPARM(3)** is 5000.

**DPARM(4) — Maximum Number of Grid Levels.**

*Input*

*On entry:* This value sets the maximum number of algebraic grid levels for the multirecursive iterative solver.

The default value of **DPARM(4)** is 10.
DPARM(5) — Dropping value for the incomplete factor. Input
On entry: This value sets the dropping value for the incomplete factorization method.

The default value of DPARM(5) is 1e-2.

DPARM(6) — Dropping value for the schurcomplement. Input
On entry: This value sets the dropping value for the schurcomplement.

The default value of DPARM(4) is 5e-3.

DPARM(7) — Maximum number of Fill-in in each column in the factor. Input
On entry: This value sets the maximum number of fill-in in each column in the factor compared to the original matrix.

The default value of DPARM(7) is 10.

DPARM(8) — Bound for the inverse of the incomplete factor $L$. Input
On entry: This value sets a numerical bound $\kappa$ for the inverse of the incomplete factor.

The default value of DPARM(8) is 500.

DPARM(9) — Maximum number of stagnation steps in Krylov-Subspace method. Input
On entry: This value sets the maximum number of non-improvement steps in Krylov-Subspace method.

The default value of DPARM(9) is 25.

DPARM(10) to DPARM(32) — Unused. Output
These values are reserved for future use. Value MUST be set to 0.

DPARM(32) — Determinant for complex unsymmetric matrices. Output
On output: This value returns the complex value of a determinant for unsymmetric matrices. The is only computed in case of IPARM(33) = 1

DPARM(33) — Determinant for real matrices. Output
On output: This value returns the determinant for real symmetric indefinite matrices. The is only computed in case of IPARM(33)=1

DPARM(34) — Relative residual after Krylov-Subspace convergence. Output
On output: This value returns the relative residual after Krylov-Subspace convergence. The is only computed in case of IPARM(32)=1

DPARM(35) — Number of Krylov-Subspace iterations. Output
On output: This value returns the number of Krylov-Subspace iterations. The is only computed in case of IPARM(32)=1

DPARM(36) to DPARM(64) — Unused. Output
These values are reserved for future use. Value MUST be set to 0.

2.5 Summary of arguments and parameters
2.6 Computing the Schur-complement: Example for nonsymmetric matrices

Suppose the system of linear equations has the form

\[
\begin{pmatrix}
A_{11} & A_{12} \\
A_{21} & A_{22}
\end{pmatrix}
\begin{pmatrix}
x_1 \\
x_2
\end{pmatrix} =
\begin{pmatrix}
b_1 \\
b_2
\end{pmatrix}.
\]

The Schur complement matrix \( S = A_{22} - A_{21}A_{11}^{-1}A_{12} \) can be returned to the user (see IPARM(39) and IPARM(39)). Pardiso provides a partial factorization of the complete matrix and returns the Schur matrix in user memory. The Schur matrix is considered as a sparse matrix and Pardiso will return matrix in sparse CSR format to the user. The partial factorization that builds the Schur matrix can also be used to solve linear systems \( A_{11}x_1 = b_1 \) associated with the \( A_{11} \) block. Entries in the right-hand side corresponding to indices from the Schur matrix need not be set on entry and they are not modified on output. The following code fragment shows how to use the Schur-complement method in Pardiso from a C++ program to compute a Schur-complement.

```c
// Reorder and partial factorization of the matrix system
int phase = 12;
iparm[38-1] = xxx; // size of the \( A_{22} \) block
pardiso(pt, &maxfct, &mnum, &mtype, &phase, &n, a, ia, ja,
         &idum, &nb, &iparm[0], &msglvl, &ddum, &ddum, &error, dparm);

// Schur-complement matrix \( S = (S, is, JS) \) in CSR format.
int nonzeros = iparm[39-1];
int* iS = new int[nrows+1];
int* JS = new int[nonzeros];
double* S = new double[nonzeros];
pardiso_get_schur(pt, &maxfct, &mnum, &mtype, S, is, JS);

// solve with \( A_{11}x_1 = b_1 \)
int phase = 33;
pardiso(pt, &maxfct, &mnum, &mtype, &phase, &n, a, ia, ja,
        &idum, &nb, &iparm[0], &msglvl, &b1, &x1, &error, dparm);
```

Table 1: Overview of subroutine arguments.
<table>
<thead>
<tr>
<th>Name</th>
<th>Type</th>
<th>Description</th>
<th>Input/Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>PT (64)</td>
<td>INT</td>
<td>Internal memory address pointer.</td>
<td>I/O</td>
</tr>
<tr>
<td>MAXFCT</td>
<td>INT</td>
<td>Number of numerical factorizations in memory.</td>
<td>I</td>
</tr>
<tr>
<td>MNUM</td>
<td>INT</td>
<td>Actual matrix to factorize.</td>
<td>I</td>
</tr>
<tr>
<td>MTYPE</td>
<td>INT</td>
<td>Matrix type.</td>
<td>I</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1: real and structurally symmetric, supernode pivoting</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>2: real and symmetric positive definite</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>-2: real and symmetric indefinite, diagonal or Bunch-Kaufman pivoting</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>11: real and nonsymmetric, complete supernode pivoting</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>3: complex and structurally symmetric, supernode pivoting</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>4: complex and hermitian positive definite</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>-4: complex and hermitian indefinite, diagonal or Bunch-Kaufman pivoting</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>6: complex and symmetric</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>13: complex and nonsymmetric, supernode pivoting</td>
<td></td>
</tr>
<tr>
<td>PHASE</td>
<td>INT</td>
<td>Solver Execution Phase.</td>
<td>I</td>
</tr>
<tr>
<td></td>
<td></td>
<td>11: Analysis</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>12: Analysis, Numerical Factorization</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>13: Analysis, Numerical Factorization, Solve, Iterative Refinement</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>22: Numerical Factorization</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>-22: Selected Inversion</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>23: Numerical Factorization, Solve, Iterative Refinement</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>33: Solve, Iterative Refinement</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>-1: Release all internal memory for all matrices</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0: Release memory for matrix number MNUM</td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>INT</td>
<td>Number of equations.</td>
<td>I</td>
</tr>
<tr>
<td>A (*)</td>
<td>R/C</td>
<td>Matrix values.</td>
<td>I</td>
</tr>
<tr>
<td>IA (N+1)</td>
<td>INT</td>
<td>Beginning of each row.</td>
<td>I</td>
</tr>
<tr>
<td>JA (*)</td>
<td>INT</td>
<td>Column indices.</td>
<td>I</td>
</tr>
<tr>
<td>PERM (N)</td>
<td>INT</td>
<td>User permutation.</td>
<td>I</td>
</tr>
<tr>
<td>NRHS</td>
<td>INT</td>
<td>Number of right-hand sides.</td>
<td>I</td>
</tr>
<tr>
<td>IPARM (64)</td>
<td>INT</td>
<td>Control parameters.</td>
<td>I/O</td>
</tr>
<tr>
<td>MSG_LVL</td>
<td>INT</td>
<td>Message level.</td>
<td>I</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0: No output.</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>1: Output statistical information</td>
<td></td>
</tr>
<tr>
<td>B (N, NRHS)</td>
<td>R/C</td>
<td>Right-hand sides.</td>
<td>I/O</td>
</tr>
<tr>
<td>X (N, NRHS)</td>
<td>R/C</td>
<td>Solution vectors (see IPARM(6)).</td>
<td>O</td>
</tr>
<tr>
<td>ERROR</td>
<td>INT</td>
<td>Error indicator.</td>
<td>O</td>
</tr>
<tr>
<td>DPARM (64)</td>
<td>REAL</td>
<td>Control parameters for iterative solver</td>
<td>I/O</td>
</tr>
</tbody>
</table>

Table 2: Overview of subroutine arguments.
<table>
<thead>
<tr>
<th>Name</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>IPARM(1)</td>
<td>Use default options.</td>
</tr>
<tr>
<td>0</td>
<td>Set all entries to their default values except IPARM(3).</td>
</tr>
<tr>
<td>IPARM(2)</td>
<td>Use METIS reordering.</td>
</tr>
<tr>
<td>0</td>
<td>Do not use METIS.</td>
</tr>
<tr>
<td>2*</td>
<td>Use METIS nested dissection reordering.</td>
</tr>
<tr>
<td>3</td>
<td>Use METIS51 nested dissection reordering.</td>
</tr>
<tr>
<td>4</td>
<td>Use AMD ordering.</td>
</tr>
<tr>
<td>IPARM(3)</td>
<td>Number of processors.</td>
</tr>
<tr>
<td>p</td>
<td>Number of OPENMP threads. This must be identical or slightly larger to the environment variable OMP_NUM_THREADS.</td>
</tr>
<tr>
<td>IPARM(4)</td>
<td>Do preconditioned CGS iterations (see description). Default is 0.</td>
</tr>
<tr>
<td>IPARM(5)</td>
<td>Use user permutation.</td>
</tr>
<tr>
<td>0*</td>
<td>Do not use user perm.</td>
</tr>
<tr>
<td>1</td>
<td>Use the permutation provided in argument PERM.</td>
</tr>
<tr>
<td>IPARM(6)</td>
<td>Solution on X / B</td>
</tr>
<tr>
<td>0*</td>
<td>Write solution to X</td>
</tr>
<tr>
<td>1</td>
<td>Write solution to B</td>
</tr>
<tr>
<td>IPARM(8)</td>
<td>Max. numbers of iterative refinement steps.</td>
</tr>
<tr>
<td>k=0*</td>
<td>Do at most k steps of iterative refinement for all matrices.</td>
</tr>
<tr>
<td>k=−99</td>
<td>No iterative refinement for all matrices.</td>
</tr>
<tr>
<td>IPARM(10)</td>
<td>eps pivot (perturbation $10^{-k}$).</td>
</tr>
<tr>
<td>13*</td>
<td>Default for nonsymmetric matrices</td>
</tr>
<tr>
<td>8*</td>
<td>Default for symmetric indefinite matrices</td>
</tr>
<tr>
<td>IPARM(11)</td>
<td>Use (non-) symmetric scaling vectors.</td>
</tr>
<tr>
<td>0</td>
<td>Do not use.</td>
</tr>
<tr>
<td>1*</td>
<td>Use (nonsymmetric matrices).</td>
</tr>
<tr>
<td>0*</td>
<td>Do not use (symmetric matrices).</td>
</tr>
<tr>
<td>IPARM(12)</td>
<td>Solve transposed matrix.</td>
</tr>
<tr>
<td>0*</td>
<td>Do normal solve.</td>
</tr>
<tr>
<td>1</td>
<td>Do solve with transposed matrix.</td>
</tr>
<tr>
<td>IPARM(13)</td>
<td>Improved accuracy using (non-)symmetric matchings</td>
</tr>
<tr>
<td>0</td>
<td>Do not use.</td>
</tr>
<tr>
<td>1*</td>
<td>Use (nonsymmetric matrices).</td>
</tr>
<tr>
<td>2</td>
<td>Use a very robust method for symmetric indefinite matrices.</td>
</tr>
<tr>
<td>0*</td>
<td>Do not use (symmetric matrices).</td>
</tr>
<tr>
<td>IPARM(18)</td>
<td>Number of nonzeros in LU.</td>
</tr>
<tr>
<td>0</td>
<td>Do not determine.</td>
</tr>
<tr>
<td>−1*</td>
<td>Will only be determined if −1 on entry.</td>
</tr>
<tr>
<td>IPARM(19)</td>
<td>Gflops for LU factorization.</td>
</tr>
<tr>
<td>0*</td>
<td>Do not determine.</td>
</tr>
<tr>
<td>−1</td>
<td>Will only be determined if −1 on entry. Increases ordering time.</td>
</tr>
</tbody>
</table>

Table 3: Overview of input IPARM control parameters. An asterisk (*) indicates the default value.
<table>
<thead>
<tr>
<th>Name</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>IPARM(21)</td>
<td>Pivoting for symmetric indefinite matrices. Default is 1.</td>
</tr>
<tr>
<td>0</td>
<td>Do sequential solve.</td>
</tr>
<tr>
<td>1*</td>
<td>Do parallel solve.</td>
</tr>
<tr>
<td>1</td>
<td>Diagonal Pivoting.</td>
</tr>
<tr>
<td>2</td>
<td>Bunch-Kaufman pivoting.</td>
</tr>
<tr>
<td>3</td>
<td>Bunch-Kaufman pivoting + rescaling before factorization.</td>
</tr>
<tr>
<td>IPARM(24)</td>
<td>Parallel Numerical Factorization</td>
</tr>
<tr>
<td>0</td>
<td>Do one-level parallel scheduling.</td>
</tr>
<tr>
<td>1*</td>
<td>Do two-level parallel scheduling.</td>
</tr>
<tr>
<td>IPARM(25)</td>
<td>Parallel Forward/Backward Solve</td>
</tr>
<tr>
<td>0</td>
<td>Do sequential solve.</td>
</tr>
<tr>
<td>1*</td>
<td>Do parallel solve.</td>
</tr>
<tr>
<td>IPARM(26)</td>
<td>Partial Forward/Backward Solve</td>
</tr>
<tr>
<td>0*</td>
<td>Do forward/backward solve with L and U.</td>
</tr>
<tr>
<td>1</td>
<td>Do forward solve with L or $U^T$.</td>
</tr>
<tr>
<td>2</td>
<td>Do backward solve with U or $L^T$.</td>
</tr>
<tr>
<td>IPARM(27)</td>
<td>Recomputation of Scaling</td>
</tr>
<tr>
<td>0*</td>
<td>Do not recompute the scaling.</td>
</tr>
<tr>
<td>1</td>
<td>Recomputation of scaling.</td>
</tr>
<tr>
<td>IPARM(28)</td>
<td>Parallel METIS reordering</td>
</tr>
<tr>
<td>0*</td>
<td>Do sequential METIS reordering.</td>
</tr>
<tr>
<td>1</td>
<td>Do parallel METIS reordering.</td>
</tr>
<tr>
<td>IPARM(29)</td>
<td>32-bit/64-bit IEEE accuracy</td>
</tr>
<tr>
<td>0*</td>
<td>Use 64-bit IEEE accuracy.</td>
</tr>
<tr>
<td>1</td>
<td>Use 32-bit IEEE accuracy.</td>
</tr>
<tr>
<td>IPARM(30)</td>
<td>Control size of supernodes</td>
</tr>
<tr>
<td>0*</td>
<td>Use default configuration.</td>
</tr>
<tr>
<td>1</td>
<td>Use use configuration.</td>
</tr>
<tr>
<td>IPARM(31)</td>
<td>Partial solve for sparse right-hand sides and sparse solutions</td>
</tr>
<tr>
<td>0*</td>
<td>Compute all components in the solution vector.</td>
</tr>
<tr>
<td>1</td>
<td>Compute only a few selected in the solution vector.</td>
</tr>
<tr>
<td>IPARM(32)</td>
<td>Use the multi-recursive iterative linear solver</td>
</tr>
<tr>
<td>0*</td>
<td>Use sparse direct solver.</td>
</tr>
<tr>
<td>1</td>
<td>Use multi-recursive iterative solver.</td>
</tr>
<tr>
<td>IPARM(33)</td>
<td>Determinant for a matrices</td>
</tr>
<tr>
<td>0*</td>
<td>Do not compute determinant.</td>
</tr>
<tr>
<td>1</td>
<td>Compute determinant.</td>
</tr>
<tr>
<td>IPARM(34)</td>
<td>Identical solution independent on the number of processors</td>
</tr>
<tr>
<td>0*</td>
<td>No identical parallel results.</td>
</tr>
<tr>
<td>1</td>
<td>Identical parallel results.</td>
</tr>
</tbody>
</table>

Table 4: Overview of input IPARM control parameters. An asterisk (*) indicates the default value.
<table>
<thead>
<tr>
<th>Name</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>IPARM(36)</td>
<td>Selected inversion for $A_{ij}^{-1}$</td>
</tr>
<tr>
<td></td>
<td>0*</td>
</tr>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>IPARM(37)</td>
<td>Selected inversion for $A_{ij}^{-1}$ for symmetric matrices</td>
</tr>
<tr>
<td></td>
<td>0*</td>
</tr>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>IPARM(38)</td>
<td>Schur-complement computation</td>
</tr>
<tr>
<td></td>
<td>0*</td>
</tr>
<tr>
<td></td>
<td>$k$</td>
</tr>
<tr>
<td>IPARM(41)</td>
<td>Incremental Update</td>
</tr>
<tr>
<td></td>
<td>0*</td>
</tr>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>IPARM(50)</td>
<td>Use out-of-core solver</td>
</tr>
<tr>
<td></td>
<td>0*</td>
</tr>
<tr>
<td></td>
<td>$&gt;0$</td>
</tr>
<tr>
<td>IPARM(53)</td>
<td>Block size of Compression</td>
</tr>
<tr>
<td></td>
<td>0*</td>
</tr>
<tr>
<td></td>
<td>$p$</td>
</tr>
<tr>
<td>IPARM(54)</td>
<td>This parameter sets the number of dense columns for compression</td>
</tr>
<tr>
<td></td>
<td>0*</td>
</tr>
<tr>
<td></td>
<td>$p$</td>
</tr>
<tr>
<td>IPARM(55)</td>
<td>METIS 4.0.1 and METIS 5.1 ordering options</td>
</tr>
<tr>
<td></td>
<td>0*</td>
</tr>
<tr>
<td></td>
<td>1</td>
</tr>
</tbody>
</table>

Table 5: Overview of input IPARM control parameters. An asterisk (*) indicates the default value.
<table>
<thead>
<tr>
<th>Name</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>IPARM(56)</td>
<td>Matching type in METIS</td>
</tr>
<tr>
<td>1</td>
<td>Random Matching (RM)</td>
</tr>
<tr>
<td>2</td>
<td>Heavy-Edge Matching (HEM, only for METIS 4.0.1).</td>
</tr>
<tr>
<td>3*</td>
<td>Sorted Heavy-Edge Matching (SHEM) (Default).</td>
</tr>
<tr>
<td>IPARM(57)</td>
<td>Algorithm used during METIS initial partitioning.</td>
</tr>
<tr>
<td>1*</td>
<td>Edge-based region growing (Default)</td>
</tr>
<tr>
<td>2</td>
<td>Node-based region growing.</td>
</tr>
<tr>
<td>IPARM(58)</td>
<td>Algorithm used for METIS refinement.</td>
</tr>
<tr>
<td>1</td>
<td>Two-sided node FM refinement.</td>
</tr>
<tr>
<td>2*</td>
<td>One-sided node FM refinement (Default).</td>
</tr>
<tr>
<td>IPARM(59)</td>
<td>Used for METIS debugging purposes.</td>
</tr>
<tr>
<td>0*</td>
<td>no output.</td>
</tr>
<tr>
<td>&gt; 0</td>
<td>output from METIS (statistics, etc, please refer to the METIS manuals).</td>
</tr>
<tr>
<td>IPARM(60)</td>
<td>Compression of the graph in METIS.</td>
</tr>
<tr>
<td>0*</td>
<td>no compression.</td>
</tr>
<tr>
<td>&gt; 0</td>
<td>compression, please refer to the METIS manuals).</td>
</tr>
<tr>
<td>IPARM(61)</td>
<td>Removal of vertices with high degree in METIS</td>
</tr>
<tr>
<td>0</td>
<td>Do not remove any vertices</td>
</tr>
<tr>
<td>p*</td>
<td>Removal of vertices</td>
</tr>
<tr>
<td>IPARM(62)</td>
<td>Trial number of separators.</td>
</tr>
<tr>
<td>1*</td>
<td>only one trial</td>
</tr>
<tr>
<td>k</td>
<td>Use k trials</td>
</tr>
</tbody>
</table>

Table 6: Overview of input IPARM control parameters. An asterisk (*) indicates the default value.
<table>
<thead>
<tr>
<th>Name</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>IPARM(7)</td>
<td>Number of iterative refinement steps.</td>
</tr>
<tr>
<td>IPARM(14)</td>
<td>Number of perturbed pivots.</td>
</tr>
<tr>
<td>IPARM(15)</td>
<td>Peak Memory in KBytes during analysis.</td>
</tr>
<tr>
<td>IPARM(16)</td>
<td>Permanent Memory in KBytes from analysis that is used in phases 2 and 3.</td>
</tr>
<tr>
<td>IPARM(17)</td>
<td>Peak Double Precision Memory in KBytes including one LU Factor.</td>
</tr>
<tr>
<td>IPARM(18)</td>
<td>Number of nonzeros in LU.</td>
</tr>
<tr>
<td></td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>−1 *</td>
</tr>
<tr>
<td>IPARM(19)</td>
<td>Gflops for LU factorization.</td>
</tr>
<tr>
<td></td>
<td>0 *</td>
</tr>
<tr>
<td></td>
<td>−1</td>
</tr>
<tr>
<td>IPARM(20)</td>
<td>Numbers of CG Iterations.</td>
</tr>
<tr>
<td>IPARM(22)</td>
<td>Number of positive eigenvalues for symmetric indefinite systems.</td>
</tr>
<tr>
<td>IPARM(23)</td>
<td>Number of negative eigenvalues for symmetric indefinite systems.</td>
</tr>
<tr>
<td>IPARM(39)</td>
<td>Number of nonzeros in Schur-complement matrix S</td>
</tr>
<tr>
<td>DPARM(32)</td>
<td>Determinant (complex value) for an unsymmetric matrices.</td>
</tr>
<tr>
<td>DPARM(33)</td>
<td>Determinant for real matrices.</td>
</tr>
<tr>
<td>DPARM(34)</td>
<td>Relative residual after Krylov-Subspace convergence.</td>
</tr>
<tr>
<td>DPARM(35)</td>
<td>Number of Krylov-Subspace iterations.</td>
</tr>
</tbody>
</table>

Table 7: Overview of output IPARM/DPARM control parameters. An asterisk (*) indicates the default value.
<table>
<thead>
<tr>
<th>Name</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>IPARM(1)</td>
<td>Use default options.</td>
</tr>
<tr>
<td>IPARM(2)</td>
<td>Fill-in reduction reordering (METIS).</td>
</tr>
<tr>
<td>IPARM(3)</td>
<td>Number of processors.</td>
</tr>
<tr>
<td>IPARM(4)</td>
<td>Preconditioned CGS / CG iterations.</td>
</tr>
<tr>
<td>IPARM(5)</td>
<td>Use user permutation.</td>
</tr>
<tr>
<td>IPARM(6)</td>
<td>Solution on X / B.</td>
</tr>
<tr>
<td>IPARM(7)</td>
<td>Executed number of iterative refinement steps.</td>
</tr>
<tr>
<td>IPARM(8)</td>
<td>Max. numbers of iterative refinement steps.</td>
</tr>
<tr>
<td>IPARM(9)</td>
<td>For future use.</td>
</tr>
<tr>
<td>IPARM(10)</td>
<td>Size of Pivot perturbation</td>
</tr>
<tr>
<td>IPARM(11)</td>
<td>Use (non-) symmetric scaling vectors.</td>
</tr>
<tr>
<td>IPARM(12)</td>
<td>Solve transposed matrix.</td>
</tr>
<tr>
<td>IPARM(13)</td>
<td>Use (non-)symmetric matchings</td>
</tr>
<tr>
<td>IPARM(14)</td>
<td>Number of perturbed pivots.</td>
</tr>
<tr>
<td>IPARM(15)</td>
<td>Peak memory [kB] phase 1.</td>
</tr>
<tr>
<td>IPARM(16)</td>
<td>Permanent integer memory [kb].</td>
</tr>
<tr>
<td>IPARM(17)</td>
<td>Peak real memory [kB] including one LU.</td>
</tr>
<tr>
<td>IPARM(18)</td>
<td>Number of nonzeros in LU.</td>
</tr>
<tr>
<td>IPARM(19)</td>
<td>Gflops for LU factorization.</td>
</tr>
<tr>
<td>IPARM(20)</td>
<td>Numbers of CG Iterations.</td>
</tr>
<tr>
<td>IPARM(21)</td>
<td>Pivoting for all matrices.</td>
</tr>
<tr>
<td>IPARM(22)</td>
<td>Number of positive eigenvalues for symmetric indefinite systems</td>
</tr>
<tr>
<td>IPARM(23)</td>
<td>Number of negative eigenvalues for symmetric indefinite systems</td>
</tr>
<tr>
<td>IPARM(24)</td>
<td>Scheduling type for numerical factorization.</td>
</tr>
<tr>
<td>IPARM(25)</td>
<td>Parallel forward/backward solve.</td>
</tr>
<tr>
<td>IPARM(26)</td>
<td>Partial forward/backward solve.</td>
</tr>
<tr>
<td>IPARM(27)</td>
<td>Recomputation of scaling.</td>
</tr>
<tr>
<td>IPARM(28)</td>
<td>Parallel METIS reordering.</td>
</tr>
<tr>
<td>IPARM(29)</td>
<td>32-bit/64-bit IEEE accuracy.</td>
</tr>
<tr>
<td>IPARM(30)</td>
<td>Size of a panel in supernodes.</td>
</tr>
<tr>
<td>IPARM(31)</td>
<td>Partial solve for sparse right-hand sides and sparse solutions.</td>
</tr>
<tr>
<td>IPARM(32)</td>
<td>Use the multi-recursive iterative linear solver.</td>
</tr>
<tr>
<td>IPARM(33)</td>
<td>Compute determinant of a real symmetric indefinite systems.</td>
</tr>
<tr>
<td>IPARM(34)</td>
<td>Identical solution independent on the number of processors.</td>
</tr>
<tr>
<td>IPARM(35)</td>
<td>Internal use.</td>
</tr>
<tr>
<td>IPARM(36)</td>
<td>Selected inversion for $A^{-1}_{ij}$.</td>
</tr>
<tr>
<td>IPARM(37)</td>
<td>Selected inversion for $A^{-1}_{ij}$ for symmetric matrices.</td>
</tr>
<tr>
<td>IPARM(38)</td>
<td>Schur-complement computation.</td>
</tr>
<tr>
<td>IPARM(39)</td>
<td>Number of nonzeros in Schur-complement matrix S.</td>
</tr>
<tr>
<td>IPARM(40)</td>
<td>Internal permutation returned to the user.</td>
</tr>
<tr>
<td>IPARM(41)</td>
<td>Incremental update.</td>
</tr>
<tr>
<td>IPARM(50)</td>
<td>Use out-of-core solver.</td>
</tr>
</tbody>
</table>

Table 8: Overview IPARM control parameters.
<table>
<thead>
<tr>
<th>Name</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>IPARM(53)</td>
<td>Block size of compression.</td>
</tr>
<tr>
<td>IPARM(54)</td>
<td>Number of dense columns for compression.</td>
</tr>
<tr>
<td>IPARM(55)</td>
<td>METIS Reordering option.</td>
</tr>
<tr>
<td>IPARM(56)</td>
<td>METIS Reordering option.</td>
</tr>
<tr>
<td>IPARM(57)</td>
<td>METIS Reordering option.</td>
</tr>
<tr>
<td>IPARM(58)</td>
<td>METIS Reordering option.</td>
</tr>
<tr>
<td>IPARM(59)</td>
<td>METIS Reordering option.</td>
</tr>
<tr>
<td>IPARM(60)</td>
<td>METIS Reordering option.</td>
</tr>
<tr>
<td>IPARM(61)</td>
<td>METIS Reordering option.</td>
</tr>
<tr>
<td>IPARM(62)</td>
<td>METIS Reordering option.</td>
</tr>
</tbody>
</table>

Table 9: Overview IPARM control parameters.
<table>
<thead>
<tr>
<th>Name</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>DPARM(1)</td>
<td>Maximum number of Krylov-Subspace Iteration.</td>
</tr>
<tr>
<td></td>
<td>300* Default. 1 ≤ DPARM(1) ≤ 500</td>
</tr>
<tr>
<td>DPARM(2)</td>
<td>Relative Residual Reduction.</td>
</tr>
<tr>
<td></td>
<td>1e − 6* Default. 1e − 1 ≥ DPARM(2) ≥ 1e − 12</td>
</tr>
<tr>
<td>DPARM(3)</td>
<td>Coarse Grid Matrix Dimension.</td>
</tr>
<tr>
<td></td>
<td>1e − 6* Default. 1e − 1 ≥ DPARM(3) ≤ 1e − 12</td>
</tr>
<tr>
<td>DPARM(4)</td>
<td>Maximum Number of Grid Levels.</td>
</tr>
<tr>
<td></td>
<td>10* Default. 1 ≤ DPARM(4) ≤ 1000</td>
</tr>
<tr>
<td>DPARM(5)</td>
<td>Dropping value for the incomplete factor.</td>
</tr>
<tr>
<td></td>
<td>1e-2* Default. 1e − 16 ≤ DPARM(5) ≤ 1</td>
</tr>
<tr>
<td>DPARM(6)</td>
<td>Dropping value for the schurcomplement.</td>
</tr>
<tr>
<td></td>
<td>5e-5* Default. 1e − 16 ≤ DPARM(6) ≤ 1</td>
</tr>
<tr>
<td>DPARM(7)</td>
<td>Maximum number of fill-in in each column in the factor.</td>
</tr>
<tr>
<td></td>
<td>10* Default. 1 ≤ DPARM(7) ≤ 100000000</td>
</tr>
<tr>
<td>DPARM(8)</td>
<td>Bound for the inverse of the incomplete factor L.</td>
</tr>
<tr>
<td></td>
<td>500* Default. 1 ≤ DPARM(9) ≤ 1000000000</td>
</tr>
<tr>
<td>DPARM(9)</td>
<td>Maximum number of non-improvement steps in Krylov-Subspace method</td>
</tr>
<tr>
<td></td>
<td>25* Default. 1 ≤ DPARM(9) ≤ 1000000000</td>
</tr>
<tr>
<td>DPARM(32)</td>
<td>Returns the determinant (complex value) for complex unsymmetric matrix.</td>
</tr>
<tr>
<td>DPARM(33)</td>
<td>Returns the determinant for a real matrix.</td>
</tr>
<tr>
<td>DPARM(34)</td>
<td>Relative residual after Krylov-Subspace convergence.</td>
</tr>
<tr>
<td>DPARM(35)</td>
<td>Number of Krylov-Subspace iterations.</td>
</tr>
</tbody>
</table>

Table 10: Overview of DPARM control parameters. An asterisk (*) indicates the default value.
3 Linking and Running the Software

In order to use Panua-Pardiso 8.0

1. Create a Panua account at https://panua.ch/authenticate/register/. If you already have an account, log in at https://panua.ch/authenticate/login/.

2. Panua-Pardiso licenses are available for Linux, Windows, and Mac (Intel & Arm).

3. Follow the instructions to generate the unique fingerprint of your machine and download the software. In order to generate the licence key, the user must supply the unique fingerprint of his machine, which is the output of the executable get_fingerprint, also available for download at http://www.panua.ch/pardiso.

4. Your license key will arrive by e-mail. Save it without any surrounding whitespace in a file called panua.lic at your home directory.

A general remark for Unix operating systems is that the environment variable LD_LIBRARY_PATH must be pointing to the Pardiso library. The environment variable OMP_NUM_THREADS must be set in all cases of using the multi-threaded library version.

3.1 LINUX on IA64 platform

The current version of Pardiso requires that either the following GNU or Intel compilers are installed on your system:

1. GNU Fortran gfortran compiler version 4.7.0 or higher for 64-bit mode.

2. GNU C compiler gcc version 4.7.0 or higher for 64-bit mode.

or

1. INTEL Fortran compiler ifort version 13.0.1 for 64-bit mode.

2. INTEL C compiler icc version 13.0.1 for 64-bit mode.

3.1.1 Linking the libraries

The libraries contain C and Fortran routines and can be linked with GNU gfortran or gcc, or Intel ifort or icc. Currently, the following library is available for LINUX IA32 and IA64 platforms:

- libpardiso.so for linking with GNU compilers version 4.7.0 on IA64 in 64-bit mode, and for linking with Intel compilers version 13.0.1 on IA64 in 64-bit mode.

Hereby follow some examples of linking user code with the Pardiso solver on 64-bit LINUX architectures. The user may need to add additional flags and options as required by the user’s code.

1. GNU version 4.6.1, 64-bit mode:

   gfortran <source/objects files> -o <executable>
   -L <Path to directory of Pardiso> -lpardiso
   -L <Path to directory of LAPACK/BLAS>
   -l <fast LAPACK and BLAS libraries> -fopenmp -lpthread -lm

   gcc <source/objects files> -o <executable>
   -L <Path to directory of Pardiso> -lpardiso
   -L <Path to directory of LAPACK/BLAS>
   -l <Fast LAPACK and BLAS libraries> -lgfortran -fopenmp -lpthread -lm
2. INTEL version 13.0.1, 64-bit mode:

  ifort <source/objects files> -o <executable>
  -L <Path to directory of Pardiso> -lpardiso
  -L <Path to directory of LAPACK/BLAS>
  -l <fast LAPACK and BLAS libraries>
  -L/opt/intel/mkl/lib/intel64 -lmkl_gf_lp64 -lmkl_sequential
  -lmkl_core -lm -lgfortran -fopenmp

  icc <source/objects files> -o <executable>
  -L <Path to directory of Pardiso> -lpardiso
  -L <Path to directory of LAPACK/BLAS>
  -l <Fast LAPACK and BLAS libraries>
  -L <Path to INTEL System libraries>
  -L/opt/intel/mkl/lib/intel64 -lmkl_gf_lp64 -lmkl_sequential
  -lmkl_core -lm -lgfortran -fopenmp

3.1.2 Setting environment

The user can control the parallel execution of the Pardiso solver with the OpenMP environment variable `OMP_NUM_THREADS`. It is very important for parallel execution that the input parameter `IPARM(3)` be set to the corresponding correct value of `OMP_NUM_THREADS`. The environment variable `LD_LIBRARY_PATH` must point to the Pardiso library. It is also recommended to set the Intel OpenMP environment variable to `MKL_SERIAL=YES`.

3.2 Mac OSX on 64-bit platforms

The current version of Pardiso requires that the following GNU compilers be installed on your system:

1. GNU C compiler gcc version 4.7.1 or higher for 64-bit mode

3.2.1 Linking the libraries

The libraries contain C and Fortran routines and can be linked with the GNU gfortran or gcc compilers. Currently, the following library is available:

- `libpardiso.dylib` for linking with GNU compilers in 64-bit mode.

Hereby follow some examples of linking user code with the Pardiso solver on Mac OSX. The user may need to add additional flags and options as required by the user’s code.

1. GNU 64-bit mode, sequential:

  gfortran <source/objects files> -o <executable>
  -L <Path to directory of Pardiso> -lpardiso.dylib
  -L <Path to directory of LAPACK/BLAS>
  -l <fast LAPACK and BLAS libraries>
  -liomp5.dylib

  gcc <source/objects files> -o <executable>
  -L <Path to directory of Pardiso> -lpardiso.dylib
  -L <Path to directory of LAPACK/BLAS>
  -l <Fast LAPACK and BLAS libraries> -liomp5.dylib

3.2.2 Setting environment

The user can control parallel execution of the Pardiso solver with the OpenMP environment variable `OMP_NUM_THREADS`. It is very important for the parallel execution that the input parameter `IPARM(3)` be set to the corresponding correct value of `OMP_NUM_THREADS`. The environment variable `LD_LIBRARY_PATH` must point to the PARDISO library.
3.3 Windows on IA64 platforms

The current version of Pardiso requires the Intel compilers and Microsoft Visual Studio 2022 compilers are installed on your system.

3.3.1 Linking the libraries

The Windows libraries comes with a library and associated import library (DLL, LIB), contain C and Fortran routines and the following libraries are available for Windows IA64 platforms:

- Parallel libpardiso.[dll, lib] for linking with Microsoft cl compilers on Intel64 in 64-bit mode.

Hereby follow some examples of linking user code with the Pardiso solver on 64-bit Windows architectures. The user may need to add additional flags and options as required by the user’s code.

1. This 64-bit Windows version comes internally with optimized BLAS/LAPACK objects from the Intel MKL library Version 10.2

   cl <source/objects files> -o <executable>
   -L <-Path to directory of Pardiso> -lpardiso.lib

4 Using the Pardiso Matrix Consistency Checker

The Pardiso 8.0 library comes with routines that can be used to check the input data for consistency. Given the input matrix $A$ and the right-hand-side vector or matrix $B$, the following tests are carried out:

- the arrays (IA, JA), that describe the nonzero structure of the matrix $A$, is a valid CSR format with index base 1
- The numerical values of $A$ and $B$, given in $A$ and $B$, are finite non-NaN values

All routines come in two flavors, one for real valued matrices and another one for complex valued matrices, which carry the suffix _Z. All arguments are exactly the same as for the main routine PARDISO, see section 2 for detailed explanation. The only output argument for all routines is ERROR. If it carries a non-zero value after calling any of the following routines, an error was found and an error message is printed to STDERR.

For testing the input matrix $A$ as described above, use the function PARDISO_CHKMATRIX:

```
SUBROUTINE PARDISO_CHKMATRIX(MTYPE, N, A, IA, JA, ERROR)
SUBROUTINE PARDISO_CHKMATRIX_Z(MTYPE, N, A, IA, JA, ERROR)
```

For testing the RHS matrix $B$ for infinite and NaN values, use the routine PARDISO_CHKVEC:

```
SUBROUTINE PARDISO_CHKVEC(N, NRHS, B, ERROR)
SUBROUTINE PARDISO_CHKVEC_Z(N, NRHS, B, ERROR)
```

A routine that automatically invokes PARDISO_CHKMATRIX and PARDISO_CHKVEC is the following:

```
SUBROUTINE PARDISO_PRINTSTATS(MTYPE, N, A, IA, JA, NRHS, B, ERROR)
SUBROUTINE PARDISO_PRINTSTATS_Z(MTYPE, N, A, IA, JA, NRHS, B, ERROR)
```

In addition to performing the above mentioned tests, this routines prints the range of absolute coefficient values to STDOUT. In many applications certain expectations for these ranges exist, so it may be useful to compare them with the printed values.

The derived C API prototypes are as follows. To represent complex valued matrices in C, please refer to the example pardiso_unsym_complex.c. In the following, we assume a structure

```
typedef struct {double re; double im;} doublecomplex;
```
We recommend to execute the checking functionality prior to every call to Pardiso with new or modified input data. Since only a few passes through the input data structures are needed in order to carry out all the tests, the induced overhead will be negligible for most applications.

5 Authors

The author Olaf Schenk (USI Lugano and Panua Technologies Sagl, olaf.schenk@panua.ch) has a list of future features that he would like to integrate into the solver. In case of errors, wishes, big surprises — it may be sometimes possible to help.

6 Acknowledgments

Functionalities related to nonsymmetric maximal matching algorithms have been implemented by Stefan Röllin (Integrated Systems Laboratory, Swiss Federal Institute of Technology, Zurich, Switzerland). We are also grateful to Esmond Ng (Lawrence Berkeley National Laboratory, Berkeley, USA) for several discussions on efficient sequential BLAS-3 supernodal factorization algorithms. We are also grateful to George Karypis (Department of Computer Science, University of Minnesota, Minnesota, USA) for his fill-in reducing METIS reordering package. Arno Liegmann (Integrated Systems Laboratory, Swiss Federal Institute of Technology, Zurich, Switzerland) provided useful help at the start of the project. We wish to thank Stefan Röllin (ETHZ), Michael Saunders (Stanford) for a providing very useful contributions to this project. The Matlab interface to PARDISO has been developed by Peter Carbonetto (UBC Vancouver). The PARDISO matrix consistency checking interface has been developed by Robert Luce (TU Berlin). Thank you all for these contributions.

7 License

You shall acknowledge using the references on page 1 References the contribution of this package in any publication of material dependent upon the use of the package. You shall use reasonable endeavors to notify the authors of the package of this publication. More information can be obtained from https://panua.ch/pardiso. In order to use Panua-Pardiso 8.0

1 Create a Panua account here https://panua.ch/authenticate/register/. If you already have an account, log in here https://panua.ch/authenticate/login/.

2 Panua-Pardiso licenses are available for Linux, Windows, and Mac (Intel & Arm).

3 Follow the instructions to generate the unique fingerprint of your machine, and download the software.

4 Your license key will arrive by e-mail. Save it without any surrounding whitespace in a file called panua.lic at your home directory.
Note that currently, Pardiso Version 8.0 also relies on the third-party code AMD ordering based on the BSD 3-clause:

AMD, Copyright (c), 1996-2015, Timothy A. Davis, Patrick R. Amestoy, and Iain S. Duff. All Rights Reserved.

PARDISO Version 8.0 uses the AMD ordering for which the following licensing conditions
Availability:

http://www.suitesparse.com

-------------------------------------------------------------------------------
AMD License: BSD 3-clause:
-------------------------------------------------------------------------------

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References


8 Further comments

Note 1: Pardiso does support since version 4.1 (January 2011) the solution of sparse symmetric indefinite systems in a message-passing parallel environment. In addition, it is a thread-safe library and the calling program can execute several parallel factorization simultaneously.

Note 2: As of version 4.0, Pardiso has out-of-core capabilities.

Note 3: Pardiso uses modified versions of METIS 4.0.1 and METIS 5.1 [2] and these versions are included in the libraries. The manual of METIS METIS 4.0.1 is available here and the user manual of METIS 5.1 is available here.

Note 4: Note that Pardiso needs the BLAS and LAPACK routines for the computational kernels. On IBM platforms these routines are part of the IBM Engineering and Scientific Subroutine Library (ESSL) and it is highly recommended that the user link with this thread-safe library. If this library is not available on your machine, then the ATLAS BLAS (http://math-atlas.sourceforge.net) or the GOTO BLAS (http://www.cs.utexas.edu/users/flame/goto) can be used. However, it is always
important that the serial and thread-safe BLAS and LAPACK library be linked because Pardiso performs all the parallelization.

Note 4: Some technical papers related to the software and the example programs in the appendices can be obtained from the site http://www.panua.ch/pardiso.

Note 5: Some remarks on the memory management of the solver. All memory is allocated in a few steps based on the true demand and at the time of the first demand. The data that is used in the symbolic analysis phase only is released at the termination of the symbolic analysis phase. The data controlling the numeric phases is reallocated. Any memory, potentially used to solve a similar linear system again, will be released only on explicit user demand by PHASE=(0, 1). Different pointer arrays PT can store data related to different structures. A small fraction of the memory is growing proportional to IPARM(3).

Pardiso memory is competing with later – with respect to the call tree – newly allocated memory in total memory size used. For large linear systems memory is likely to be the limiting issue, hence the user code should follow the strategy 'allocate as late and free as soon as possible', too.

A Examples for sparse symmetric linear systems

Appendices A.2 and A.3 give two examples (Fortran, C) for solving symmetric linear systems with PARDISO. They solve the system of equations $Ax = b$, where

$$
A = \begin{pmatrix}
7.0 & 0.0 & 1.0 & 0.0 & 0.0 & 2.0 & 7.0 & 0.0 \\
0.0 & -4.0 & 8.0 & 0.0 & 2.0 & 0.0 & 0.0 & 0.0 \\
1.0 & 8.0 & 1.0 & 0.0 & 0.0 & 0.0 & 0.0 & 5.0 \\
0.0 & 0.0 & 0.0 & 7.0 & 0.0 & 0.0 & 9.0 & 0.0 \\
0.0 & 2.0 & 0.0 & 0.0 & 5.0 & -1.0 & 5.0 & 0.0 \\
2.0 & 0.0 & 0.0 & 0.0 & -1.0 & 0.0 & 0.0 & 5.0 \\
7.0 & 0.0 & 0.0 & 9.0 & 5.0 & 0.0 & 11.0 & 0.0 \\
0.0 & 0.0 & 5.0 & 0.0 & 0.0 & 5.0 & 0.0 & 5.0 \\
\end{pmatrix}
\quad \text{and} \quad
b = \begin{pmatrix}
1.0 \\
2.0 \\
3.0 \\
4.0 \\
5.0 \\
6.0 \\
7.0 \\
8.0 \\
\end{pmatrix}.$$

A.1 Example results for symmetric systems

Upon successful execution of the solver, the result of the solution $X$ is as follows:

[PARDISO]: License check was successful ...
[PARDISO]: Matrix type : real symmetric
[PARDISO]: Matrix dimension : 8
[PARDISO]: Matrix non-zeros : 18
[PARDISO]: Abs. coeff. range: min 0.00e+00 max 1.10e+01
[PARDISO]: RHS no. 1: min 0.00e+00 max 7.00e+00

============= PARDISO: solving a symmetric indef. system =============

Summary PARDISO 8.0: ( reorder to reorder )

=============

Times:

========

Time fulladj: 0.000000 s
Time reorder: 0.000000 s
Time symbfct: 0.000000 s
Time parlist: 0.000000 s
Time malloc : 0.000000 s
Statistics:

### Parallel Direct Factorization with #cores: 1
### and #nodes: 1
### Numerical Factorization with Level-3 BLAS performance

### Linear system $Ax = b$
- #equations: 8
- #non-zeros in A: 18
- non-zeros in A (%): 28.125000
- #right-hand sides: 1

### Factors L and U
- #columns for each panel: 80
- # of independent subgraphs: 0

### Preprocessing with state of the art partitioning metis
- #supernodes: 5
- size of largest supernode: 4
- number of nonzeros in L 29
- number of nonzeros in U 1
- number of nonzeros in L+U 30
- number of perturbed pivots 0
- number of nodes in solve 8
- mflop for the numerical factorization: 0.000075

Reordering completed ...
Number of nonzeros in factors = 30
Number of factorization MFLOPS = 0

summary PARDISO 8.0: ( factorize to factorize )

Times:

Time A to LU: 0.000000 s
Time numfct : 0.000000 s
Time malloc : 0.000000 s
Time total : 0.000000 s total - sum: 0.000000 s

Statistics:

### Parallel Direct Factorization with #cores: 1
### and #nodes: 1
### Numerical Factorization with Level-3 BLAS performance

### Linear system $Ax = b$
- #equations: 8
- #non-zeros in A: 18
- non-zeros in A (%): 28.125000
Factorization completed ...

=============== PARDISO: solving a symmetric indef. system ===============

Summary PARDISO 8.0: (solve to solve)

Times:

Time solve : 0.000000 s
Time total : 0.000000 s total - sum: 0.000000 s

Statistics:

< Parallel Direct Factorization with #cores: > 1
< and #nodes: > 1
< Numerical Factorization with Level-3 BLAS performance >

< Linear system Ax = b>
#equations: 8
#non-zeros in A: 18
non-zeros in A (%): 28.125000
#right-hand sides: 1

< Factors L and U >
#columns for each panel: 80
# of independent subgraphs: 0

< preprocessing with state of the art partitioning metis>
#supernodes: 5
size of largest supernode: 4
number of nonzeros in L 29
number of nonzeros in U 1
number of nonzeros in L+U 30
number of perturbed pivots 0
number of nodes in solve 8
mflop for the numerical factorization: 0.000075
Solve completed ...
The solution of the system is:
x(1) = -0.3168031420208231
x(2) = -0.4955685649709140
x(3) = -0.2129608358961057
x(4) = 5.6704583348771778E-002
x(5) = 0.8607062136425950
x(6) = 0.3140983363592574
x(7) = 0.4003408796176218
x(8) = 1.4988624995368485

A.2 Example pardiso_sym.f for symmetric linear systems (including selected inversion)

************************************************************************************************
! Copyright (C) 2022 until present, by Panua Technologies Sagl, Switzerland. All Rights Reserved.
!
! Example program to show the use of the "PARDISO" routine for symmetric linear systems
!
! This program can be downloaded from the following site: http://www.panua.ch
! The above copyright and permission notice must appear in all partial and complete
! copies of this file.
************************************************************************************************

PROGRAM pardiso_sym
IMPLICIT NONE

!.. Internal solver memory pointer
INTEGER*8 pt(64)

!.. All other variables
INTEGER maxfct, mnum, mtype, phase, n, nrhs, error, msglvl
INTEGER iparm(64)
INTEGER ia(9)
INTEGER ja(18)
REAL*8 dparm(64)
REAL*8 a(18)
REAL*8 b(8)
REAL*8 x(8)
REAL*8 y(8)

INTEGER i, j, idum, solver
REAL*8 waltime1, waltime2, ddum, normb, normr

!.. Fill all arrays containing matrix data.

DATA n /8/, nrhs /1/, maxfct /1/, mnum /1/
DATA ia /1,5,8,10,12,15,17,18,19/
DATA ja

1 1
2 2, 3, 5
DATA a
1 /7.d0, 1.d0, 2.d0, 7.d0,
2 -4.d0, 8.d0, 2.d0,
3 1.d0, 5.d0,
4 7.d0, 9.d0,
5 5.d0, 1.d0, 5.d0,
6 0.d0, 5.d0,
7 11.d0,
8 5.d0/

! .. set right hand side
do i = 1, n
   b(i) = 1.d0
end do

! .. Setup Pardiso control parameters and initialize the solvers
!   internal adress pointers. This is only necessary for the FIRST
!   call of the PARDISO solver.

mtype = -2 ! unsymmetric matrix symmetric, indefinite
solver = 10 ! use sparse direct method

! .. PARDISO license check and initialize solver
call pardisoinit(pt, mtype, solver, iparm, dparm, error)

! .. Numbers of Processors (value of OMP_NUM_THREADS)
iparm(3) = 1

IF (error .NE. 0) THEN
   IF (error.EQ.-10 ) WRITE(*,*), 'No license file found'
   IF (error.EQ.-11 ) WRITE(*,*), 'License is expired'
   IF (error.EQ.-12 ) WRITE(*,*), 'Wrong username or hostname'
   STOP
ELSE
   WRITE(*,*), '[PARDISO]: License check was successful ... '
END IF

! .. pardiso_chk_matrix(...)  
! Checks the consistency of the given matrix.
! Use this functionality only for debugging purposes
CALL pardiso_chkmatrix (mtype, n, a, ia, ja, error);
IF (error .NE. 0) THEN
   WRITE(*,*), 'The following ERROR was detected: ', error
   STOP
ENDIF

! .. pardiso_chkvec(...)
! Checks the given vectors for infinite and NaN values
! Input parameters (see PARDISO user manual for a description):
! Use this functionality only for debugging purposes
CALL pardiso_chkvec (n, nrhs, b, error);
IF (error .NE. 0) THEN
    WRITE(*,*) 'The following ERROR was detected: ', error
    STOP
ENDIF

!.. pardiso_printstats(...)!
! prints information on the matrix to STDOUT.
! Use this functionality only for debugging purposes
CALL pardiso_printstats (mtype, n, a, ia, ja, nrhs, b, error);
IF (error .NE. 0) THEN
    WRITE(*,*) 'The following ERROR was detected: ', error
    STOP
ENDIF

!.. Reordering and Symbolic Factorization, This step also allocates
! all memory that is necessary for the factorization

phase = 11 ! only reordering and symbolic factorization
msglvl = 1 ! with statistical information
iparm(33) = 1 ! compute determinant

CALL pardiso (pt, maxfct, mnum, mtype, phase, n, a, ia, ja,
1     idum, nrhs, iparm, msglvl, ddum, ddum, error, dparm)
WRITE(*,*) 'Reordering completed ... '
IF (error .NE. 0) THEN
    WRITE(*,*) 'The following ERROR was detected: ', error
    STOP
ENDIF
WRITE(*,*) 'Number of nonzeros in factors = ',iparm(18)
WRITE(*,*) 'Number of factorization MFLOPS = ',iparm(19)
!.. Factorization.
phase = 22 ! only factorization
CALL pardiso (pt, maxfct, mnum, mtype, phase, n, a, ia, ja,
1     idum, nrhs, iparm, msglvl, ddum, ddum, error, dparm)
IF (iparm(33).EQ.1) THEN
    write(*,*) 'Log of determinant is ', dparm(33)
ENDIF
WRITE(*,*) 'Factorization completed ... '
IF (error .NE. 0) THEN
    WRITE(*,*) 'The following ERROR was detected: ', error
    STOP
ENDIF
!.. Back substitution and iterative refinement
   iparm(8) = 1  ! max numbers of iterative refinement steps
   phase = 33  ! only solve

   CALL pardiso (pt, maxfct, mnum, mtype, phase, n, a, ia, ja,
   1     idum, nrhs, iparm, msglvl, b, x, error, dparm)

   WRITE(*,*), 'Solve completed ... '

   WRITE(*,*), 'The solution of the system is '
   DO i = 1, n
      WRITE(*,*), 'x(',i,') = ', x(i)
   END DO

!.. Residual test
   normb = 0
   normr = 0
   CALL pardiso_residual (mtype, n, a, ia, ja, b, x, y, normb, normr)
   WRITE(*,*), 'The norm of the residual is ', normr/normb

!.. Selected Inversion

   phase = -22
   CALL pardiso (pt, maxfct, mnum, mtype, phase, n, a, ia, ja,
   1     idum, nrhs, iparm, msglvl, b, x, error, dparm)

!.. Print diagonal elements of the inverse of A = (a, ia, ja) */
   do i = 1, n
      j = ia(i)
      WRITE(*,*), 'Diagonal',i,'-element of A^{-1} = ', a(j)
   end do

!.. Termination and release of memory

   phase = -1  ! release internal memory
   CALL pardiso (pt, maxfct, mnum, mtype, phase, n, idum, idum, idum,
   1     idum, nrhs, iparm, msglvl, idum, ddum, error, dparm)

END

A.3 Example pardiso_sym.c for symmetric linear systems (including selected inversion)

/*******************************************************************************/
Copyright (C) 2022 until present, by Panua Technologies Sagl, Switzerland. All Rights Reserved.

Example program to show the use of the "PARDISO" routine on symmetric linear systems.

This program can be downloaded from the following site: http://www.panua.ch
The above copyright and permission notice must appear in all partial and complete copies of this file.
*******************************************************************************/
```c
#include <stdio.h>
#include <stdlib.h>
#include <math.h>

/* PARDISO prototype. */
void pardisoinit (void *, int *, int *, int *, double *, int *);
void pardiso (void *, int *, int *, int *, int *, int *,
               double *, int *, int *, int *, int *, int *,
               int *, double *, double *, int *, double *);
void pardiso_chkmatrix (int *, int *, double *, int *, int *, int *);
void pardiso_chkvec (int *, int *, double *, int *, int *);
void pardiso_printstats (int *, int *, double *, int *, int *, int *,
                         double *, int *);

int main( void )
{
    /* Matrix data. */
    int n = 8;
    int ia[9] = { 0, 4, 7, 9, 11, 14, 16, 17, 18 };
    int ja[18] = { 0, 2, 4, 5, 6,
                   1, 2, 4,
                   2, 7,
                   3, 6,
                   4, 5, 6,
                   5, 7,
                   6, 7 };
    double a[18] = { 7.0, 1.0, 2.0, 7.0,
                     -4.0, 8.0, 2.0,
                     1.0, 5.0, 7.0,
                     9.0,
                     5.0, 1.0, 5.0,
                     0.0, 5.0,
                     11.0,
                     5.0 };
    int nnz = ia[n];
    int mtype = -2; /* Real symmetric matrix */

    /* RHS and solution vectors. */
    double b[8], x[8]; /* Number of right hand sides. */
    int nrhs = 1;

    /* Internal solver memory pointer pt, */
    /* 32-bit: int pt[64]; 64-bit: long int pt[64] */
    /* or void *pt[64] should be OK on both architectures */
    void *pt[64];

    /* Pardiso control parameters. */
    int iparm[64];
```
double dparm[64];
int maxfct, mnum, phase, error, msglvl, solver;

/* Number of processors. */
int num_procs;

/* Auxiliary variables. */
char *var;
int i, k;

double ddum;  /* Double dummy */
int idum;  /* Integer dummy. */

/* ------------------------------- */
/* .. Setup Pardiso control parameters. */
/* ------------------------------- */

error = 0;
solver=0; /* use sparse direct solver */
pardisoinit (pt, &mtype, &solver, iparm, dparm, &error);

if (error != 0)
{
    if (error == -10 )
        printf("No license file found \n");
    if (error == -11 )
        printf("License is expired \n");
    if (error == -12 )
        printf("Wrong username or hostname \n");
    return 1;
}
else
    printf("[PARDISO]: License check was successful ... \n");

/* Numbers of processors, value of OMP_NUM_THREADS */
var = getenv("OMP_NUM_THREADS");
if(var != NULL)
    sscanf( var, "%d", &num_procs );
else {
    printf("Set environment OMP_NUM_THREADS to 1");
    exit(1);
}
iparm[2] = num_procs;

maxfct = 1;  /* Maximum number of numerical factorizations. */
mnum = 1;  /* Which factorization to use. */
msglvl = 1;  /* Print statistical information */
error = 0;  /* Initialize error flag */

/* ------------------------------- */
/* .. Convert matrix from 0-based C-notation to Fortran 1-based */
/* notation. */
for (i = 0; i < n+1; i++) {
    ia[i] += 1;
}
for (i = 0; i < nnz; i++) {
    ja[i] += 1;
}

/* Set right hand side to i. */
for (i = 0; i < n; i++) {
    b[i] = i;
}

pardiso_chkmatrix (&mtype, &n, a, ia, ja, &error);
if (error != 0) {
    printf("ERROR in consistency of matrix: %d", error);
    exit(1);
}

pardiso_chkvec (&n, &nrhs, b, &error);
if (error != 0) {
    printf("ERROR in right hand side: %d", error);
    exit(1);
}

pardiso_printstats (&mtype, &n, a, ia, ja, &nrhs, b, &error);
if (error != 0) {
    printf("ERROR right hand side: %d", error);
    exit(1);
}

/* Reordering and Symbolic Factorization. This step also allocates */
/* all memory that is necessary for the factorization. */
phase = 11;

pardiso (pt, &maxfct, &mnum, &mtype, &phase,
      &n, a, ia, ja, &idum, &nrhs,
      iparm, &msglvl, &ddum, &ddum, &error, dparm);

if (error != 0) {
    printf("\nERROR during symbolic factorization: %d", error);
    exit(1);
}
printf("\nReordering completed ... ");
printf("\nNumber of nonzeros in factors = %d", iparm[17]);
printf("\nNumber of factorization MFLOPS = %d", iparm[18]);

/* -------------------------------------------------------------------- */
/* ... Numerical factorization. */
/* -------------------------------------------------------------------- */

phase = 22;
iparm[32] = 1; /* compute determinant */

pardiso (pt, &maxfct, &mnum, &mtype, &phase,
      &n, a, ia, ja, &idum, &nrhs,
      iparm, &msglvl, &ddum, &ddum, &error, dparm);

if (error != 0) {
    printf("\nERROR during numerical factorization: %d", error);
    exit(2);
}
printf("\nFactorization completed ...
");

/* -------------------------------------------------------------------- */
/* ... Back substitution and iterative refinement. */
/* -------------------------------------------------------------------- */

phase = 33;

iparm[7] = 1; /* Max numbers of iterative refinement steps. */

pardiso (pt, &maxfct, &mnum, &mtype, &phase,
      &n, a, ia, ja, &idum, &nrhs,
      iparm, &msglvl, b, x, &error, dparm);

if (error != 0) {
    printf("\nERROR during solution: %d", error);
    exit(3);
}
printf("\nSolve completed ...

");
printf("\nThe solution of the system is: ");
for (i = 0; i < n; i++) {
    printf("\n x [%d] = % f", i, x[i] );
}
p printf ("\n\n");
/* ... Inverse factorization. */
if (solver == 0)
{
    printf("Compute Diagonal Elements of the inverse of A ...
    phase = -22;
    iparm[35] = 1; /* no not overwrite internal factor L */
    pardiso (pt, &maxfct, &mnum, &mtype, &phase, &n, ia, ja, &idum, &nrhs,
             iparm, &msglvl, b, x, &error, dparm);
    /* print diagonal elements */
    for (k = 0; k < n; k++)
    {
        int j = ia[k]-1;
        printf ("Diagonal element of A^{-1} = %d %d %32.24e
", k, ja[j]-1, a[j]);
    }
}

/* ... Convert matrix back to 0-based C-notation. */
for (i = 0; i < n+1; i++) {
    ia[i] -= 1;
}
for (i = 0; i < nnz; i++) {
    ja[i] -= 1;
}

/* ... Termination and release of memory. */
phase = -1; /* Release internal memory. */
 pardiso (pt, &maxfct, &mnum, &mtype, &phase,
           &n, &ddum, ia, ja, &idum, &nrhs,
           iparm, &msglvl, &ddum, &ddum, &error, dparm);
 return 0;
}

A.4 Example pardiso_sym_schur.cpp for Schur-complement computation for symmetric linear systems

***********************************************************************************************
Copyright (C) 2022 until present, by Panua Technologies Sagl, Switzerland. All Rights Reserved.
Example program to show the use of the "PARDISO" routine for evaluating the Schur-complement.
This program can be downloaded from the following site: http://www.panua.ch
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// C++ compatible
#include <fstream>
#include <iostream>
#include <iomanip>
#include <cmath>
#include <stdlib.h>
using namespace std;

/* PARDISO prototype. */
extern "C" void pardisoinit_d(void *, int *, int *, int *, double *, int *);
extern "C" void pardiso_d(void *, int *, int *, int *, int *, int *,
            double *, int *, int *, int *, int *,
            int *, double *, double *, int *, double *);
extern "C" void pardiso_chkmatrix_d(int *, int *, double *, int *, int *, int *);
extern "C" void pardiso_chkvec_d(int *, int *, double *, int *);
extern "C" void pardiso_printstats_d(int *, int *, double *, int *, int *,
            double *, int *, double *, int *, int *, double *);
extern "C" void pardiso_get_schur_d(void*, int*, int*, int*, double*, int*, int*);

void dumpCSR(const char* filename, int n, int* ia, int* ja, double* a)
{
    fstream fout(filename, ios::out);
    fout << n << endl;
    fout << n << endl;
    fout << ia[n] << endl;
    for (int i = 0; i <= n; i++)
    {
        fout << ia[i] << endl;
    }
    for (int i = 0; i < ia[n]; i++)
    {
        fout << ja[i] << endl;
    }
    for (int i = 0; i < ia[n]; i++)
    {
        fout << a[i] << endl;
    }
    fout.close();
}

void printCSR(int n, int nnz, int* ia, int* ja, double* a)
{
    cout << "rows: " << setw(10) << n << "n" << endl;
}
cout << "nnz : " << setw(10) << nnz << endl;

if (nnz == n*n)
{
    for (int i = 0; i < n; i++)
    {
        for (int j = 0; j < n; j++)
        {
            cout << setw(10) << a[i*n + j];
        }
        cout << endl;
    }
}
else
{
    for (int i = 0; i < n; i++)
    {
        for (int index = ia[i]; index < ia[i+1]; index++)
        {
            int j = ja[index];
            cout << setw(10) << "(" << i << ", " << j << ") " << a[index];
        }
        cout << endl;
    }
}

void shiftIndices(int n, int nonzeros, int* ia, int* ja, int value)
{
    int i;
    for (i = 0; i < n+1; i++)
    {
        ia[i] += value;
    }
    for (i = 0; i < nonzeros; i++)
    {
        ja[i] += value;
    }
}

int main( void )
{
    /* Matrix data. */
    int n =10;
    int ia[11] = { 0, 6, 11, 15, 19, 22, 26, 29, 32, 33, 34};
    int ja[34] = { 0, 2, 5, 6, 8, 9, 1, 2, 4, 8, 9, 2, 7, 8, 9, 3, 6, 8, 9, 4, 8, 9, 5, 7, 8, 9, 6, 8, 9, 7, 8, 9, 8, 9};
}
```c
double a[34] = { 7.0, 1.0, 2.0, 7.0, 17.0, 8.5,
                 -4.0, 8.0, 2.0, 6.0, 3.0,
                 1.0, 5.0, 6.0, 3.0,
                 7.0, 9.0, 16.0, 8.0,
                 0.0, -4.0,-2.0,
                 3.0, 8.0,18.0, 9.0,
                 11.0, 12.0, 6.0,
                 5.0, 4.0, 2.0,
                 0.0,
                 0.0};

int nnz = ia[n];
int mtype = -2;  /* Real unsymmetric matrix */

dumpCSR("matrix_sample_mtype_2.csr", n, ia, ja, a);
printCSR(n, nnz, ia, ja, a);

/* Internal solver memory pointer pt, */
/* 32-bit: int pt[64]; 64-bit: long int pt[64] */
/* or void *pt[64] should be OK on both architectures */
void *pt[64];

/* Pardiso control parameters. */
int iparm[65];
double dparm[64];
int solver;
int maxfct, mnum, phase, error, msglvl;

/* Number of processors. */
int num_procs;

/* Auxiliary variables. */
char *var;
int i;

double ddum;  /* Double dummy */
int idum;    /* Integer dummy. */

/* ..................................................................... */
/* ... Setup Pardiso control parameters and initialize the solvers */
/* internal address pointers. This is only necessary for the FIRST */
/* call of the PARDISO solver. */
/* .....................................................................*/

error = 0;
solver = 0; /* use sparse direct solver */
pardisoinit_d(pt, &mtype, &solver, &iparm[1], dparm, &error);

if (error != 0)
{
    if (error == -10 )
```
printf("No license file found \n");
if (error == -11)
    printf("License is expired \n");
if (error == -12)
    printf("Wrong username or hostname \n");
return 1;
else
    printf("[PARDISO]: License check was successful ... \n");

/* Numbers of processors, value of OMP_NUM_THREADS */
var = getenv("OMP_NUM_THREADS");
if(var != NULL)
    sscanf(var, "%d", &num_procs);
else{
    printf("Set environment OMP_NUM_THREADS to 1");
    exit(1);
}
iparm[3] = num_procs;
iparm[13] = 0;

maxfct = 1; /* Maximum number of numerical factorizations. */
mnum = 1; /* Which factorization to use. */
msglvl = 1; /* Print statistical information */
error = 0; /* Initialize error flag */

/* -------------------------------------------------------------------- */
/* .. Convert matrix from 0-based C-notation to Fortran 1-based */
/* notation. */
/* -------------------------------------------------------------------- */
shiftIndices(n, nnz, ia, ja, 1);

/* -------------------------------------------------------------------- */
/* .. pardiso_chk_matrix(...) */
/* Checks the consistency of the given matrix. */
/* Use this functionality only for debugging purposes */
/* -------------------------------------------------------------------- */
pardiso_chkmatrix_d(&mtype, &n, a, ia, ja, &error);
if (error != 0)
{
    printf("\nERROR in consistency of matrix: %d", error);
    exit(1);
}

/* -------------------------------------------------------------------- */
/* .. Reordering and Symbolic Factorization. This step also allocates */
/* all memory that is necessary for the factorization. */
/* -------------------------------------------------------------------- */

int nrows_S = 2;
phase = 12;
iparm[38] = nrows_S;

int nb = 0;
pardiso_d(pt, &maxfct, &mnum, &mtype, &phase,
&n, a, ia, ja, &idum, &nb,
&iparm[1], &msglvl, &ddum, &ddum, &error, dparm);

if (error != 0)
{
    printf("\nERROR during symbolic factorization: %d", error);
    exit(1);
}
printf("\nReordering completed ...
");
printf("Number of nonzeros in factors = %d", iparm[18]);
printf("Number of factorization MFLOPS = %d", iparm[19]);
printf("Number of nonzeros is S = %d", iparm[39]);

/* -------------------------------------------------------------------- */
/* .. allocate memory for the Schur-complement and copy it there. */
/* -------------------------------------------------------------------- */

int nonzeros_S = iparm[39];

int* iS = new int[nrows_S+1];
int* jS = new int[nonzeros_S];
double* S = new double[nonzeros_S];
pardiso_get_schur_d(pt, &maxfct, &mnum, &mtype, S, iS, jS);

/* -------------------------------------------------------------------- */
/* .. Convert matrix from 1-based Fortan notation to 0-based C */
/* notation. */
/* -------------------------------------------------------------------- */

shiftIndices(n, nnz, ia, ja, -1);

/* -------------------------------------------------------------------- */
/* .. Convert Schur complement from Fortran notation held internally */
/* to 0-based C notation */
/* -------------------------------------------------------------------- */

shiftIndices(nrows_S, nonzeros_S, iS, jS, -1);

printCSR(nrows_S, nonzeros_S, iS, jS, S);

/* -------------------------------------------------------------------- */
/* .. Termination and release of memory. */
/* -------------------------------------------------------------------- */

phase = -1; /* Release internal memory. */
A.5 Example iterative solver pardiso_iter_sym.f for symmetric linear systems

!************************************************************************************************
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!
! Example program to show the use of the "PARDISO" routine for symmetric linear systems.
!
! This program can be downloaded from the following site: http://www.panua.ch
!
! The above copyright and permission notice must appear in all partial and complete
! copies of this file.
!************************************************************************************************

PROGRAM pardiso_sym
IMPLICIT NONE

!.. Internal solver memory pointer for 64-bit architectures
!.. INTEGER*8 pt(64)
!.. Internal solver memory pointer for 32-bit architectures
!.. INTEGER*4 pt(64)
!.. This is OK in both cases
INTEGER*8 pt(64)

!.. All other variables
INTEGER maxfct, mnum, mtype, phase, n, nrhs, error, msglvl
INTEGER iparm(64)
INTEGER ia(9)
INTEGER ja(18)
REAL*8 dparm(64)
REAL*8 a(18)
REAL*8 b(8)
REAL*8 x(8)

INTEGER i, idum, solver
REAL*8 waltime1, waltime2, ddum

!.. Fill all arrays containing matrix data.
DATA n /8/, nrhs /1/, maxfct /1/, mnum /1/
DATA ia /1,5,8,10,12,15,17,18,19/
DATA ja
  1   /1,  3,  6,  7,  
  2       2,  3,  5,  
  3       3,  8,  
  4             4,  7,  
  5       5,  6,  7,  
  6             6,  8,  
  7       7,  
  8                      8/
DATA a
  1 /7.d0,  1.d0,  2.d0,  7.d0,  
  2       -4.d0,  8.d0,  2.d0,  
  3       1.d0,  5.d0,  
  4             7.d0,  9.d0,  
  5       5.d0,  1.d0,  5.d0,  
  6             0.d0,  5.d0,  
  7       11.d0,  
  8                      5.d0/

! .. set right hand side
  do i = 1, n
    b(i) = 1.d0
  end do

! .. Setup Pardiso control parameters und initialize the solvers
! internal adress pointers. This is only necessary for the FIRST
! call of the PARDISO solver.

  mtype = -2 ! unsymmetric matrix symmetric, indefinite
  solver = 1 ! use sparse direct method

! .. pardiso_chk_matrix(...)  
! Checks the consistency of the given matrix.
! Use this functionality only for debugging purposes
CALL pardiso_chkmatrix (mtype, n, a, ia, ja, error);
IF (error .NE. 0) THEN
  WRITE(*,*) 'The following ERROR was detected: ', error
  STOP
ENDIF

! .. pardiso_chkvec(...)  
! Checks the given vectors for infinite and NaN values
! Input parameters (see PARDISO user manual for a description):
! Use this functionality only for debugging purposes
CALL pardiso_chkvec (n, nrhs, b, error);
IF (error .NE. 0) THEN
  WRITE(*,*) 'The following ERROR was detected: ', error
  STOP
! .. pardiso_printstats(...)  
! prints information on the matrix to STDOUT.  
! Use this functionality only for debugging purposes

CALL pardiso_printstats (mtype, n, a, ia, ja, nrhs, b, error);  
IF (error .NE. 0) THEN  
    WRITE(*,*)'The following ERROR was detected: ', error  
    STOP  
ENDIF

! .. Pardiso license check and initialize solver
CALL pardisoinit (pt, mtype, solver, iparm, dparm, error)  
IF (error .NE. 0) THEN  
    IF (error.EQ.-10 ) WRITE(*,*)'No license file found'  
    IF (error.EQ.-11 ) WRITE(*,*)'License is expired'  
    IF (error.EQ.-12 ) WRITE(*,*)'Wrong username or hostname'  
    STOP  
ELSE  
    WRITE(*,*)'[PARDISO]: License check was successful ... '  
ENDIF

! .. Numbers of Processors (value of OMP_NUM_THREADS)
iparm(3) = 1

phase = 12  ! set up of the preconditioner
msglvl = 1

CALL pardiso (pt, maxfct, mnum, mtype, phase, n, a, ia, ja,  
1       idum, nrhs, iparm, msglvl, ddum, ddum, error, dparm)

WRITE(*,*)'Set up completed ... '
IF (error .NE. 0) THEN  
    WRITE(*,*)'The following ERROR was detected: ', error  
    STOP  
ENDIF

! .. Solve with SQMR
phase = 33  ! only solve

! .. Set initial solution to zero
DO i = 1, n
    x(i) = 0
END DO

CALL pardiso (pt, maxfct, mnum, mtype, phase, n, a, ia, ja,  
1       idum, nrhs, iparm, msglvl, b, x, error, dparm)

WRITE(*,*)'Solve completed ... '
WRITE(*,*)'The solution of the system is '
DO i = 1, n
    WRITE(*,*)'x(',i,') = ', x(i)
END DO

!.. Termination and release of memory
phase = -1 ! release internal memory
CALL pardiso (pt, maxfct, mnum, mtype, phase, n, ddum, idum, idum, idum, idum, idum, nrhs, iparm, msglvl, ddum, ddum, error, dparm)
END

B Release notes

PARDISO - Release Notes - version 8.0
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PARDISO Contents
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PARDISO - a direct and iterative sparse linear solver library - is a
tuned math solver library designed for high performance on homogeneous
multicore machines and cluster of multicores including Intel Xeon, Intel
Itanium, AMD Opteron, and IBM Power processors, and includes both 32-bit
and 64-bit library versions. Different versions are available for Linux
MAC OSX, and Windows 64-bit operating systems.

A full suite of sparse direct linear solver routines for all kind of
sparse matrices is provided, and key datastructures have been designed
for high performance on various multicore processors and cluster of multicores
processors in 64-bit modes.

A selected suite of iterative linear solvers for real and complex symmetric
indefinite matrices is provided that takes advantage of a new algebraic
multilevel incomplete factorization that is especially designed for good
performance in very large-scale applications.

New Features
------------
New features of release PARDISO 8.0:
(o) METIS 5.1 is now integrated with internal symbols.
(o) Various performance improvements for Ipopt.

New features of release PARDISO 7.2:
(o) Several important bug fixes.
(o) Fast incremental updates.
(o) Several performance updates for circuit matrices.

New features of release PARDISO 7.0:
(o) Added incremental LU updates, multiple-rank parallel update algorithms for
    nonsymmetric sparse LU factors. The incremental update been been proven to be
    very useful for transient simulations in circuit simulation.
(o) Added much faster internal block factorization method.

New features of release PARDISO 6.3:
(o) Added AMD reordering
(o) Added user control for METIS 4.0.1 and METIS 5.1
New features of release PARDISO 6.1:

(o) Added acceleration of block orderings for symmetric indefinite matrices.
(o) Significantly improved the reordering time for matrices including dense columns.

New features of release PARDISO 6.0:

(o) Added METIS 5.1 as additional preprocessing method.
(o) Improved scalability for factorization on higher number of cores.
(o) Added out-of-core option for real and complex symmetric indefinite matrices.
(o) New internal data structure to simplify future developments.

New features of release PARDISO 5.0.0:

(o) Switch to host-free license for all 64-bit libraries. This allows all users to use the PARDISO software within a cluster environment.
(o) Full support of multi-threaded Schur-complement computations for all kind of matrices.
(o) Full support of multi-threaded parallel selected inversion to compute selected entries of the inverse of A.
(o) Faster multi-threaded code for the solution of multiple right hand sides.
(o) Full support of 32-bit sequential and parallel factorizations for all kind of matrices.

New features of release PARDISO 4.1.3:

(o) Bug fix in the computation of the log of the determinant.

New features of release PARDISO 4.1.2:

(o) Support of different licensing types
   - Evaluation license for academic and commercial use.
   - Academic license (host-unlocked, user-locked, 1 year).
   - Commercial single-user license (host-unlocked, user-locked, 1 year).
   - Commercial license (host-unlocked, user-unlocked, redistributable, 1 year).

New features of release PARDISO 4.1.0:

(o) New host-unlimited licensing mechanism integrated into PARDISO 4.1.0.
   We now have two different options available:
   - a time-limited user-host locked license, and
   - a time-limited user-locked (host-free) license.

(o) 32-bit sequential and parallel factorization and solve routines for real unsymmetric matrices (matrix_type = 11). Mixed-precision refinement can used for these 32-bit sparse direct factorizations.

(o) Additional routines that can check the input data (matrix, right-hand-side) (contribution from Robert Luce, TU Berlin)

New features of release 4.0.0 of PARDISO since version 3.3.0:

(o) Due to the new features the interface to PARDISO and PARDISOINIT has changed! This version is not backward compatible!
Reproducibility of exact numerical results on multi-core architectures. The solver is now able to compute the exact bit identical solution independent on the number of cores without effecting the scalability. Here are some results for a nonlinear FE model with 500'000 elements.

Intel MKL PARDISO 10.2
1 core - factor: 17.980 sec., solve: 1.13 sec.
4 cores - factor: 6.120 sec., solve: 1.05 sec.
8 cores - factor: 3.830 sec., solve: 1.05 sec.

U Basel PARDISO 4.0.0:
1 core - factor: 16.820 sec., solve: 1.09 sec.
2 cores - factor: 9.021 sec., solve: 0.67 sec.
4 cores - factor: 5.186 sec., solve: 0.53 sec.
8 cores - factor: 3.170 sec., solve: 0.43 sec.

This method is currently only working for symmetric indefinite matrices.

32-bit sequential and parallel factorization and solve routines for real symmetric indefinite matrices, for symmetric complex matrices and for structurally symmetric matrices. Mixed-precision refinement is used for these 32-bit sparse direct factorizations.

Internal 64-bit integer datastructures for the numerical factors allow to solve very large sparse matrices with over $2^{32}$ nonzeros in the sparse direct factors.

Work has been done to significantly improve the parallel performance of the sparse direct solver which results in a much better scalability for the numerical factorization and solve on multicore machines. At the same time, the workspace memory requirements have been substantially reduced, making the PARDISO direct routine better able to deal with large problem sizes.

Integration of a parallel multi-threaded METIS reordering that helps to accelerate the reordering phase (Done by to Stefan Roellin, ETH Zurich)

Integration of a highly efficient preconditioning method that is based on a multi-recursive incomplete factorization scheme and stabilized with a new graph-pivoting algorithm. The method have been selected by the SIAM Journal of Scientific Computing as a very important milestone in the area of new solvers for symmetric indefinite matrices and the related paper appeared as a SIGEST SIAM Paper in 2008.

This preconditioner is highly effective for large-scale matrices with millions of equations.

(o) Support of 32-bit and 64-bit Windows operating systems (based on
Intel Professional Compiler Suite and the Intel MKL Performance
Library)

(o) A new extended interface to direct and iterative solver.
   Double-precision parameters are passed by a dparm array to the solver.
   The interface allow for greater flexibility in the storage of input
   and output data within supplied arrays through the setting of increment
   arguments.

(o) Note that the interface to PARDISO and PARDISOINIT has changed and that
   this version is not backward compatible.

(o) Computation of the determinant for symmetric indefinite matrices.

(o) Solve A^Tx=b using the factorization of A.

(o) The solution process e.g. LUX=b can be performed in several phases that
   the user can control.

(o) This version of PARDISO is compatible with the interior-point optimization
   package IPOPT version 3.7.0

(o) A new matlab interface has been added that allows a flexible use of all
   direct and iterative solvers.

Contributions
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The following colleagues have contributed to the solver (in alphabetical order):

Peter Carbonetto (UBC Vancouver, Canada)
Radim Janalik (USI Lugano, Switzerland)
George Karypis (U Minnesota, US)
Arno Liegmann (ETHZ, Switzerland)
Esmond Ng (LANL, US)
Stefan Roellin (ETHZ, Switzerland)
Michael Saunders (Stanford, US)

Applicability
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Different PARDISO versions are provided for use with the GNU compilers
gfortran/gcc, with the Intel ifort compiler, and (for use under Solaris)
with the Sun f95/cc compilers.

Required runtime libraries under Microsoft Windows
-------------------------------------------------------
PARDISO version 4.0.0 and later link with the standard runtime library
provided by the Microsoft Visual Studio 2008 compilers. This requires
that the machine PARDISO runs on either has VS2K8 installed (or the Windows
SDK for Windows Server 2008), or the runtime libraries can be separately
downloaded from the appropriate Microsoft platform links provided below:
Visual Studio 2K8 Redist:

x86

x64

Bug Reports
------------
Bugs should be reported to contact@panua.ch with the string "PARDISO-Support" in the subject line.